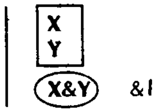
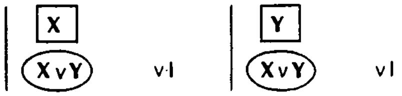
PRIMITIVE SENTENCE LOGIC RULES FOR NATURAL DEDUCTION

Conjunction Introduction



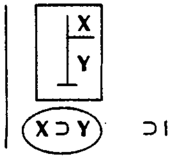
Conjunction Elimination

Disjunction Introduction

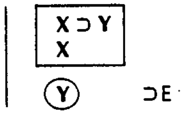


Disjunction Elimination

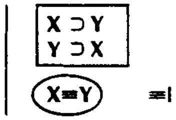
Conditional Introduction



Conditional Elimination

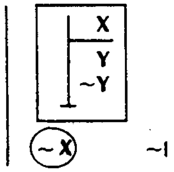


Biconditional Introduction

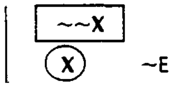


Biconditional Elimination

Negation Introduction



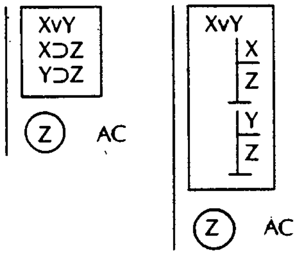
Negation Elimination



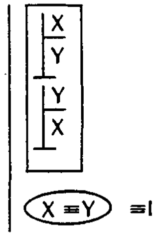
Reiteration: If a sentence occurs, either as a premise or as a conclusion in a derivation, that sentence may be copied (reiterated) in any of that derivation's LOWER sub-derivations, or lower down in the same derivation.

DERIVED SENTENCE LOGIC RULES FOR NATURAL DEDUCTION

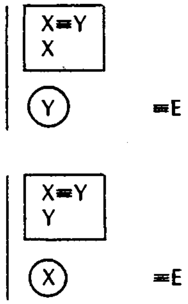
Argument by Cases



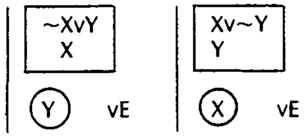
Biconditional Introduction



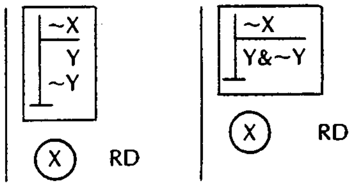
Biconditional Elimination



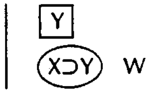
Disjunction Elimination



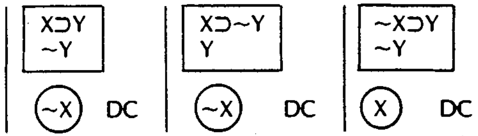
Reductio Ad Absurdum



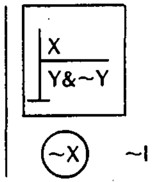
Weakening



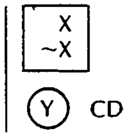
Denying the Consequent



Negation Introduction



Contradiction



De Morgan's Rules

~(XvY) and ~X&~Y are mutually derivable (DM)
~(X&Y) and ~Xv~Y are mutually derivable (DM)

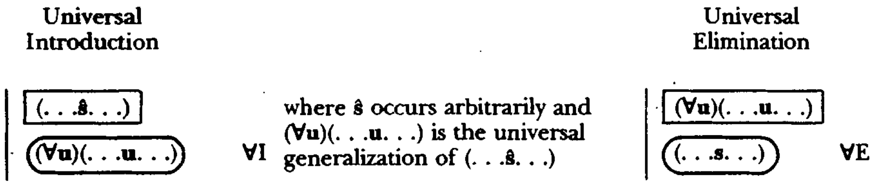
Contraposition

X⊃Y and ~Y⊃~X are mutually derivable (CP)
~X⊃Y and ~Y⊃X are mutually derivable (CP)
X⊃~Y and Y⊃~X are mutually derivable (CP)

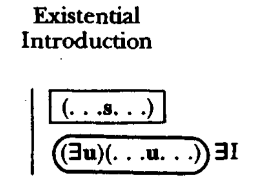
Conditional Rules

X⊃Y and ~XvY are mutually derivable (C)
~(X⊃Y) and X&~Y are mutually derivable (C)

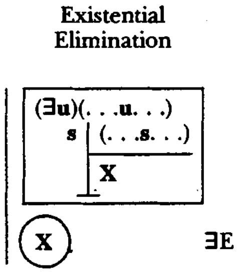
PRIMITIVE PREDICATE LOGIC RULES FOR NATURAL DEDUCTION



where ŝ occurs arbitrarily and (∀u)(. . .u. . .) is the universal generalization of (. . .ŝ. . .)

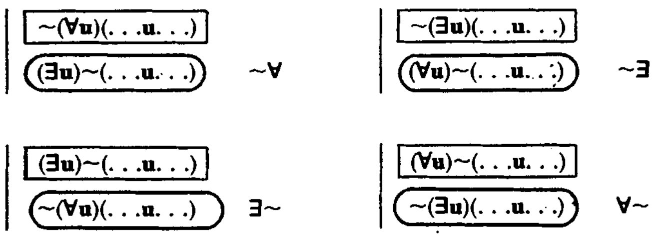


Where (∃u)(. . .u. . .) is an existential generalization of (. . .s. . .)

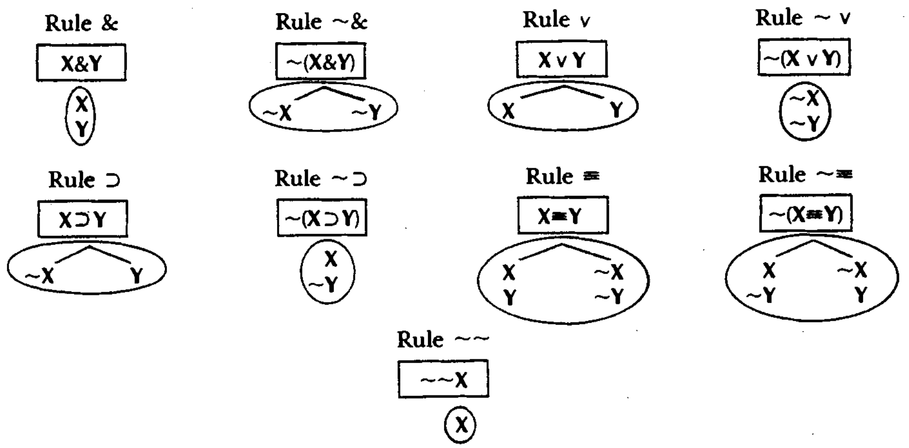


Where (. . .s. . .) is a substitution instance of (∃u)(. . .u. . .) and s is isolated in the subderivation

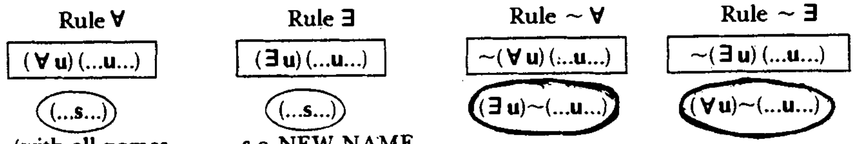
DERIVED RULES FOR NEGATED QUANTIFIERS



SENTENCE LOGIC TRUTH TREE RULES



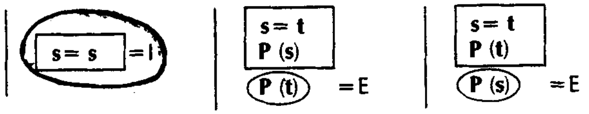
PREDICATE LOGIC TRUTH TREE RULES



(with all names, s, on the branch)

s a NEW NAME

NATURAL DEDUCTION RULES FOR IDENTITY AND FUNCTION SYMBOLS



Function symbols: Treat all constant terms alike in applying ∀E and ∃I. Apply ∀I and ∃E only to names.

TRUTH TREE RULES FOR IDENTITY AND FUNCTION SYMBOLS

Rule ≠: Close any branch on which s ≠ s appears.

Rule =: When s = t appears on a branch, substitute s and t for each other wherever possible, without checking the resulting lines.

For function symbols: Instantiate all universally quantified sentences with all constant terms on the branch. In existentially quantified sentences use only one NEW NAME.