Natural Deduction for Sentence Logic 5

Fundamentals

5-1. THE IDEA OF NATURAL DEDUCTION

In chapter 4 you learned that saying an argument is valid means that any case which makes all of the argument's premises true also makes its conclusion true. And you learned how to test for validity by using truth tables, by exhaustively checking all the relevant cases, that is, all the lines of the truth table. But truth tables are horribly awkward. It would be nice to have a way to check validity which looked more like the forms of argument we know from everyday life.

Natural deduction does just that. When we speak informally, we use many kinds of valid arguments. (I'll give some examples in a moment.) Natural deduction makes these familiar forms of argument exact. It also organizes them in a system of valid arguments in which we can represent absolutely any valid argument.

Let's look at some simple and, I hope, familiar forms of argument. Suppose I know (say, because I know Adam's character) that if Adam loves Eve, then he will ask Eve to marry him. I then find out from Adam's best friend that Adam does indeed love Eve. Being a bright fellow, I immediately conclude that a proposal is in the offing. In so doing I have used the form of argument traditionally called 'modus ponens', but which I am going to call *Conditional Elimination*.

Conditional Elimination

$$\begin{array}{ll} \mathbf{X \supset Y} & \\ \underline{\mathbf{X}\qquad\quad} & \supset E \\ \mathbf{Y} & \end{array}$$

Logicians call such an argument form a *Rule of Inference*. If, in the course of an argument, you are given as premises (or you have already concluded) a sentence of the form **X⊃Y** and the sentence **X,** you may draw as a conclusion the sentence **Y.** This is because, as you can check with a truth table, in any case in which sentences of the form **X⊃Y** and **X** are both true, the sentence **Y** will be true also. You may notice that I have stated these facts, not for some particular sentences 'A⊃B', 'A', and 'B', but for sentence forms expressed with boldfaced '**X**' and '**Y**'. This is to emphasize the fact that this form of argument is valid no matter what specific sentences might occur in the place of '**X**' and '**Y**'.

Here is another example of a very simple and common argument form, or rule of inference:

Disjunction Elimination

$$\begin{array}{ll} \mathbf{X \vee Y} & \\ \underline{\mathbf{\sim X}\qquad\quad} & \vee E \\ \mathbf{Y} & \end{array}$$

If I know that either Eve will marry Adam or she will marry no one, and I then somehow establish that she will not marry Adam (perhaps Adam has promised himself to another), I can conclude that Eve will marry no one. (Sorry, even in a logic text not all love stories end happily!) Once again, as a truth table will show, this form of argument is valid no matter what sentences occur in the place of '**X**' and in the place of '**Y**'.

Though you may never have stopped explicitly to formulate such rules of argument, all of us use rules like these. When we argue we also do more complicated things. We often give longer chains of argument which start from some premises and then repeatedly use rules in a series of steps. We apply a rule to premises to get an intermediate conclusion. And then, having established the intermediate conclusion, we can use it (often together with some of the other original premises) to draw further conclusions.

Let's look at an example to illustrate how this process works. Suppose you are given the sentences 'A⊃B', 'B⊃C', and 'A' as premises. You are asked to show that from these premises the conclusion 'C' follows. How can you do this?

It's not too hard. From the premises 'A⊃B' and 'A', the rule of conditional elimination immediately allows you to infer 'B':

$$\begin{array}{ll} A \supset B & \\ \underline{A\quad\;} & \supset E \\ B & \end{array}$$

But now you have 'B' available in addition to the original premise 'B⊃C'. From these two sentences, the rule of conditional elimination allows you to infer the desired conclusion 'C':

$$\begin{array}{ll} B \supset C & \\ \underline{B\quad\;} & \supset E \\ C & \end{array}$$

I hope this example is easy to follow. But if I tried to write out an example with seven steps in this format, things would get impossibly confusing. We need a streamlined way of writing chains of argument.

The basic idea is very simple. We begin by writing all our premises and then drawing a line to separate them from the conclusions which follow. But now we allow ourselves to write any number of conclusions below the line, as long as the conclusions follow from the premises. With some further details, which I'll explain in a minute, the last example looks like this:

1	A⊃B	*P*
2	B⊃C	*P*
3	A	*P*
4	B	1, 3, ⊃E
5	C	2, 4, ⊃E

Lines 1 through 5 constitute a *Derivation* of conclusions 4 and 5 from premises 1, 2, and 3. In thinking about such a derivation, you should keep most clearly in mind the idea that the conclusions are supposed to follow from the premises, in the following sense: Any assignment of truth values to sentence letters which makes the premises all true will also make all of the conclusions true.

In a derivation, every sentence below the horizontal line follows from the premises above the line. But sentences below the line may follow directly or indirectly. A sentence follows directly from the premises if a rule of inference applies directly to premises to allow you to draw the sentence as a conclusion. This is the way I obtained line 4. A sentence follows indirectly from the premises if a rule of inference applies to some conclusion already obtained (and possibly also to an original premise) to allow you to draw the sentence as a conclusion. The notation on the right tells you that the first three sentences are premises. It tells you that line 4 is *Licensed* (i.e., permitted) by applying the rule of conditional elimination to the sentence of lines 1 and 3. And the notation for line 5 tells you that line 5 is licensed by applying the rule of conditional elimination to the sentences of lines 2 and 4.

For the moment don't worry too much about the vertical line on the left. It's called a *Scope Line*. Roughly speaking, the scope line shows what hangs together as one extended chain of argument. You will see why scope lines are useful when we introduce a new idea in the next section.

You should be sure you understand why it is legitimate to draw conclusions indirectly from premises, by appealing to previous conclusions. Again, what we want to guarantee is that any case (i.e., any assignment of truth values to sentence letters) which makes the premises true will also make each of the conclusions true. We design the rules of inference so that whenever they apply to sentences and these sentences happen to be true, then the conclusion licensed by the rule will be true also. For short, we say that the rules are *Truth Preserving*.

Suppose we have a case in which all of the premises are true. We apply a rule to some of the premises, and because the rule is truth preserving, the conclusion it licenses will, in our present case, also be true. (Line 4 in the last example illustrates this.) But if we again apply a rule, this time to our first conclusion (and possibly some premise), we are again applying a rule to sentences which are, in the present case, all true. So the further conclusion licensed by the rule will be true too. (As an illustration, look at line 5 in the last example.) In this way, we see that if we start with a case in which all the premises are true and use only truth preserving rules, all the sentences which follow in this manner will be true also.

To practice, let's try another example. We'll need a new rule:

Disjunction Introduction

X	vI
XvY	

which says that if **X** is true, then so is **XvY.** If you recall the truth table definition of 'v', you will see that disjunction introduction is a correct, truth preserving rule of inference. The truth of even one of the disjuncts in a disjunction is enough to make the whole disjunction true. So if **X** is true, then so is **XvY,** whatever the truth value of **Y**.

Let's apply this new rule, together with our two previous rules, to show that from the premises 'A⊃~B', 'BvC', and 'A', we can draw the conclusion 'CvD'. But first close the book and see if you can do it for yourself.

The derivation looks like this:

1	A⊃~B	P
2	BvC	P
3	A	P
4	~B	1, 3, ⊃E
5	C	2, 4, vE
6	CvD	5, vI

The sentence of line 4 (I'll just say "line 4" for short) is licensed by applying conditional elimination to lines 1 and 3. Line 5 is licensed by applying disjunction elimination to lines 2 and 4. Finally, I license line 6 by applying disjunction introduction to line 5.

EXERCISES

5–1. For each of the following arguments, provide a derivation which shows the argument to be valid. That is, for each argument construct a derivation which uses as premises the argument's premises and which has as final conclusion the conclusion of the argument. Be sure to number and annotate each step as I have done with the examples in the text. That is, for each conclusion, list the rule which licenses drawing the conclusion and the line numbers of the sentences to which the rule applies.

a) ~P⊃~D, ~D⊃~F, ~P / ~F

b) ~C⊃~D, ~C / ~DvE

c) Fv~G, ~F, GvK / K

d) A⊃B, A, B⊃~C, CvD / DvE

e) Lv~M, ~L, MvD, D⊃H / H

f) C, C⊃(HvA), ~H / Av~K

g) (Kv~D)⊃F, K / FvD

h) D, (DvB)⊃~G, (~Gv~H)⊃(GvQ) / Qv~A

i) (Mv~T)⊃(AvJ), ~A, BvM, ~A⊃~B / JvD

5-2. SUBDERIVATIONS

Many of you have probably been thinking: So far, we have an "introduction" and an "elimination" rule for disjunction and just an "elimination" rule for the conditional. I bet that by the time we're done we will have exactly one introduction and one elimination rule for each connective. That's exactly right. Our next job is to present the introduction rule for the conditional, which involves a new idea.

How can we license a conclusion of the form **X⊃Y?** Although we could do this in many ways, we want to stick closely to argument forms from everyday life. And most commonly we establish a conclusion of the form **X⊃Y** by presenting an **argument** with **X** as the premise and **Y** as the conclusion. For example, I might be trying to convince you that if Adam loves Eve, then Adam will marry Eve. I could do this by starting from the assumption that Adam loves Eve and arguing, on that assumption, that matrimony will ensue. Altogether, I will not have shown that Adam and Eve will get married, because in my argument I used the unargued assumption that Adam loves Eve. But I will have shown that **if** Adam loves Eve, then Adam will marry Eve.

Let's fill out this example a bit. Suppose that you are willing to grant, as premises, that if Adam loves Eve, Adam will propose to Eve ('A⊃B'), and that if Adam proposes, marriage will ensue ('B⊃C'). But neither you nor I have any idea whether or not Adam does love Eve (whether 'A' is true). For the sake of argument, let's add to our premises the temporary assumption, 'A', which says that Adam loves Eve, and see what follows. Assuming 'A', that Adam loves Eve, we can conclude 'B' which says that Adam will propose (by conditional elimination, since we have as a premise 'A⊃B', that if Adam loves Eve, he will propose). And from the conclusion 'B', that Adam will propose, we can further conclude 'C', that marriage will ensue (again by conditional elimination, this time appealing to the premise 'B⊃C', that proposal will be followed by marriage). So, on the temporary assumption 'A', that Adam loves Eve, we can conclude 'C', that marriage will ensue. But the assumption was only temporary. We are not at all sure that it is true, and we just wanted to see what would follow from it. So we need to discharge the temporary assumption, that is, restate what we can conclude from our permanent premises without making the temporary assumption. What is this? Simply 'A⊃C', that if Adam loves Eve, marriage will ensue.

Presenting this example in English takes a lot of words, but the idea is in fact quite simple. Again, we badly need a streamlined means of representing what is going on. In outline, we have shown that we can establish a conditional of the form **X⊃Y** not on the basis of some premises (or not from premises alone), but on the strength of an argument. We need to write down the argument we used, and, after the **whole argument,** write down the sentence which the argument establishes. We do it like this:


```
3 |   | A
  |   |----
4 |   | A⊃B
5 |   | B⊃C
6 |   | B      3, 4, ⊃E
7 |   | C      5, 6, ⊃E
  |
8 | A⊃C        3-7, Conditional Introduction (⊃I)
```

For right now, don't worry about where lines 4 and 5 came from. Focus on the idea that lines 3 through 7 constitute an entire argument, which we call a *Subderivation,* and the conclusion on line 8 follows from the fact that we have validly derived 'C' from 'A'. A subderivation is always an integral part of a larger, or *Outer Derivation.* Now you can see why I have been using the vertical scope lines. We must keep outer derivations and subderivations separated. A continuous line to the left of a series of sentences indicates to you what pieces hang together as a derivation. A derivation may have premises, conclusions, **and** subderivations, which are full-fledged derivations in their own right.

A subderivation can provide the justification for a new line in the outer derivation. For the other rules we have learned, a new line was justified by applying a rule to one or two prior **lines.** Our new rule, conditional introduction (⊃I), justifies a new line, 8 in our example, by appealing to a **whole subderivation,** 3–7 in our example. When a rule applies to two prior lines, we list the line numbers separated by commas—in the example line 6 is licensed by applying ⊃E to lines 3 and 4. But when we justify a new line (8 in our example) by applying a rule (here, ⊃I) to a whole subderivation, we cite the whole subderivation by writing down its inclusive lines numbers (3–7 in our example).

Now, where did lines 4 and 5 come from in the example, and why did I start numbering lines with 3? I am trying to represent the informal example about Adam and Eve, which started with the real premises that if Adam loves Eve, Adam will propose (A⊃B), and that if Adam proposes, they will marry (B⊃C). These are premises in the original, outer derivation, and I am free to use them anywhere in the following argument, including in any subderivation which forms part of the main argument. Thus the whole derivation looks like this:

```
1 | A⊃B           P
2 | B⊃C           P
  |----
3 |    | A        Assumption (A)
  |    |----
4 |    | A⊃B      1, Reiteration (R)
5 |    | B⊃C      2, Reiteration (R)
6 |    | B        3, 4, ⊃E
7 |    | C        5, 6, ⊃E
8 | A⊃C           3-7, Conditional Introduction (⊃I)
```

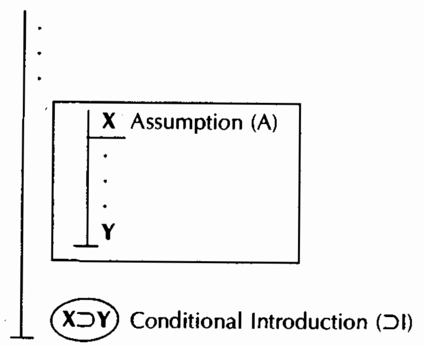
I am licensed to enter lines 4 and 5 in the subderivation by the rule:

> *Reiteration:* If a sentence occurs, either as a premise or as a conclusion in a derivation, that sentence may be copied (reiterated) in any of that derivation's **lower** subderivations, or lower down in the same derivation.

In the present example, 'A⊃B' and 'B⊃C' are assumed as premises of the whole argument, which means that everything that is supposed to follow is shown to be true **only** on the assumption that these original premises are true. Thus we are free to assume the truth of the original premises anywhere in our total argument. Furthermore, if we have already shown that something follows from our original premises, this conclusion will be true whenever the original premises are true. Thus, in any following subderivation, we are free to use any conclusions already drawn.

At last I can give you the full statement of what got us started on this long example: the rule of *Conditional Introduction.* We have been looking only at a very special example. The same line of thought applies whatever the details of the subderivation. In the following schematic presentation, what you see in the box is what you must have in order to apply the rule of conditional introduction. You are licensed to apply the rule when you see something which has the form of what is in the box. What you see in the circle is the conclusion which the rule licenses you to draw.

Conditional Introduction



> In words: If you have, as part of an outer derivation, a subderivation with assumption **X** and final conclusion **Y**, then **X⊃Y** may be entered below the subderivation as a further conclusion of the outer derivation. The subderivation may use any previous premise or conclusion of the outer derivation, entering these with the reiteration rule.

You will have noticed that the initial sentences being assumed in an outer, or main, derivation get called "premises," while the initially as-

sumed sentence in a subderivation gets called an "assumption." This is because the point of introducing premises and assumptions is slightly different. While we are arguing, we appeal to premises and assumptions in exactly the same way. But premises always stay there. The final conclusion of the outer derivation is guaranteed to be true only in those cases in which the premises are true. But an assumption introduced in a subderivation gets *Discharged.*

This is just a new word for what we have been illustrating. The point of the subderivation, beginning with assumption **X** and ending with final conclusion **Y,** is to establish **X⊃Y** as part of the outer derivation. Once the conclusion, **X⊃Y,** has been established and the subderivation has been ended, we say that the assumption, **X,** has been discharged. In other words, the scope line which marks the subderivation signals that we may use the subderivation's special assumption only within that subderivation. Once we have ended the subderivation (indicated with the small stroke at the bottom of the vertical line), we are not, in the outer derivation, subject to the restriction that **X** is assumed to be true. If the premises of the original derivation are true, **X⊃Y** will be true whether **X** is true or not.

It's very important that you understand why this last statement is correct, for understanding this amounts to understanding why the rule for conditional introduction works. Before reading on, see if you can state for yourself why, if the premises of the original derivation are true, and there is a subderivation from **X** as assumption to **Y** as conclusion, **X⊃Y** will be true whether or not **X** is true.

The key is the truth table definition of **X⊃Y.** If **X** is false, **X⊃Y** is, by definition, true, whatever the truth value of **Y.** So we only have to worry about cases in which **X** is true. If **X** is true, then for **X⊃Y** to be true, we need **Y** to be true also. But this is just what the subderivation shows: that for cases in which **X** is true, **Y** is also true. Of course, if the subderivation used premises from the outer derivation or used conclusions that followed from those premises, the subderivation only shows that in all cases in which **X and** the original premises are true, **Y** will also be true. But then we have shown that **X⊃Y** is true, not in absolutely all cases, but in at least those cases in which the original premises are true. But that's just right, since we are entering **X⊃Y** as a conclusion of the outer derivation, subject to the truth of the original premises.

EXERCISES

5–2. Again, for each of the following arguments, provide a derivation which shows the argument to be valid. Be sure to number and annotate each step to show its justification. All of these exercises will require you to use conditional introduction and possibly other of the rules you have already learned. You may find the use of conditional introduction difficult until you get accustomed to it. If so, don't be alarmed, we're going to work on it a lot. For these problems you will find the following strategy very helpful: If the final conclusion which you are trying to derive (the "target conclusion") is a conditional, set up a subderivation which has as its assumption the antecedent of the target conclusion. That is, start your outer derivation by listing the initial premises. Then start a subderivation with the target conclusion's antecedent as its assumption. Then reiterate your original premises in the subderivation and use them, together with the subderivation's assumptions, to derive the consequent of the target conclusion. If you succeed in doing this, the rule of conditional introduction licenses drawing the target conclusion as your final conclusion of the outer derivation.

a) A⊃B, B⊃C, C⊃D / A⊃D

b) NvP / ~N⊃P

c) B / A⊃B

d) ~B / (BvC)⊃C

e) K⊃~D, DvH / K⊃H

f) A⊃B / A⊃(BvC)

g) F⊃(CvM), ~C / F⊃M

h) (DvB)⊃J / D⊃J

i) A⊃K, (KvP)⊃L / A⊃L

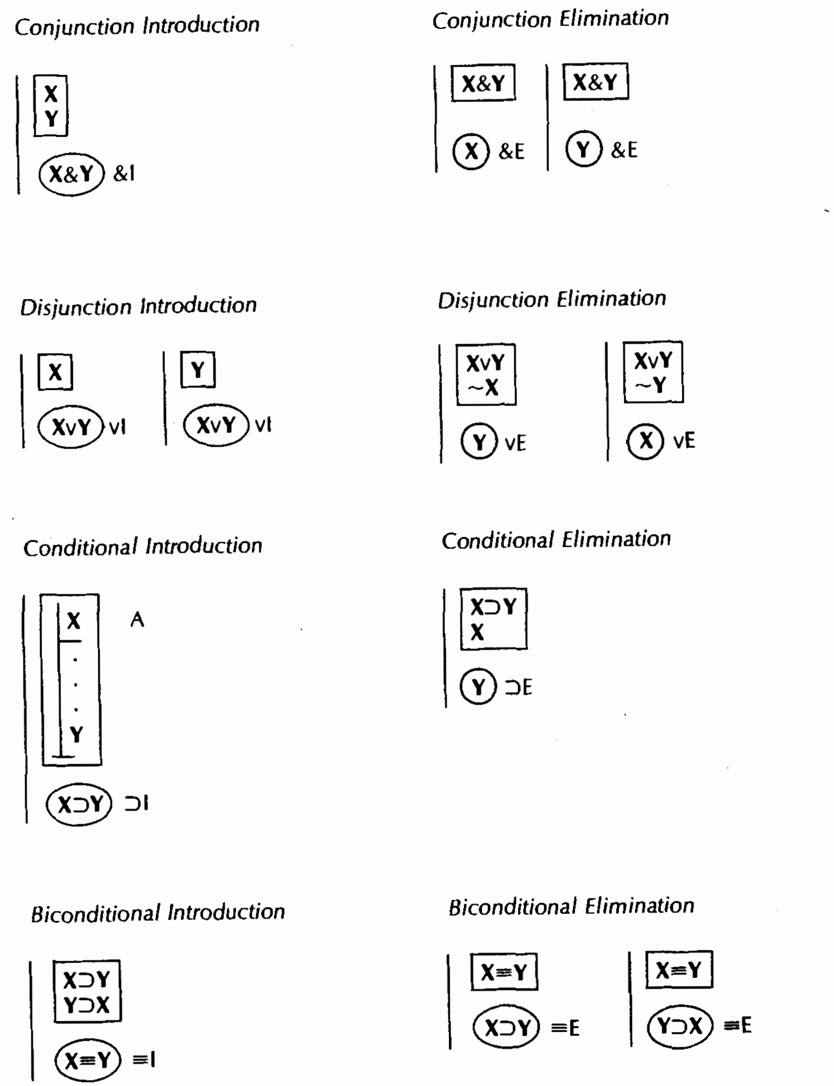
j) Q⊃~S, Q⊃(SvF) / Q⊃F

k) P, (~DvK)⊃B, (Fv~D)⊃~K, P⊃(Kv~F) / (Fv~D)⊃(Bv~P)

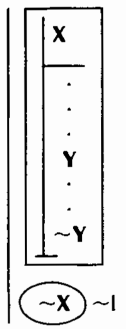
5-3. THE COMPLETE RULES OF INFERENCE

We now have in place all the basic ideas of natural deduction. We need only to complete the rules. So that you will have them all in one place for easy reference, I will simply state them all in abbreviated form and then comment on the new ones. Also, I will now state all of the rules using the same format. For each rule I will show a schematic derivation with one part in a box and another part in a circle. In the box you will find, depending on the rule, either one or two sentence forms or a subderivation

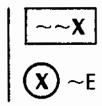
form. In the circle you will find a sentence form. To apply a given rule in an actual derivation, you proceed as follows: You look to see whether the derivation has something with the same form as what's in the box. If so, the rule licenses you to write down, as a new conclusion, a sentence with the form of what's in the circle.



Negation Introduction



Negation Elimination



> *Reiteration:* If a sentence occurs, either as a premise or as a conclusion in a derivation, that sentence may be copied (reiterated) in any of that derivation's **lower** subderivations, or lower down in the same derivation.

In interpreting these schematic statements of the rules, you must remember the following: When a rule applies to two sentences, as in the case of conjunction introduction, the two sentences can occur in either order, and they may be separated by other sentences. The sentences to which a rule applies may be premises, an assumption, or prior conclusions, always of the same derivation, that is, lying along the same scope line. Also, the sentence which a rule licenses you to draw may be written anywhere below the licensing sentences or derivation, but as part of the same derivation, again, along the same scope line.

Conjunction introduction and elimination are so simple we rarely bother to mention them when we argue informally. But to be rigorous and complete, our system must state and use them explicitly. Conjunction introduction states that when two sentences, **X** and **Y**, appear in a derivation, in either order and whether or not separated by other sentences, we may conclude their conjunction, **X&Y**, anywhere below the two conjuncts. Conjunction elimination just tells us that if a conjunction of the form **X&Y** appears on a derivation, we may write either conjunct (or both, on different lines) anywhere lower down on the derivation. We have already discussed the rules for disjunction and the conditional. Here we need only add that in the elimination rules, the sentences to which the rules apply may occur in either order and may be separated by other sentences. For example, when applying disjunction elimination, the rule applies to sentences of the form **XvY** and **~X**, in whatever order those sentences occur and whether or not other sentences appear between them.

Biconditional introduction and elimination really just express the fact that a biconditional of the form **X≡Y** is logically equivalent to the conjunction of sentences of the form **X⊃Y** and **Y⊃X**. If the two conditionals appear on a derivation, whatever the order, and whether or not separated by other sentences, we may write the biconditional lower down as a conclusion. Conversely, if a biconditional of the form **X≡Y** appears, one may write lower down, as a conclusion, **X⊃Y**, **Y⊃X**, or both (on separate lines).

Note that negation elimination licenses dropping a **double** negation, and is justified by the fact that **X** is always logically equivalent to **~~X**.

Negation introduction requires some comment. Once again, natural deduction seeks to capture and make precise conventional forms of informal argument. This time we express what traditionally goes under the name of "reductio ad absurdum," or "reduction to the absurd." Here the idea is that if we begin with an assumption from which we can deduce a contradiction, the original assumption must be false. Natural deduction employs this strategy as follows: Begin a subderivation with an assumption, **X**. If one succeeds in deriving both a sentence of the form **Y** and its negation, **~Y**, write the sentence of the form **~X** as a conclusion of the outer derivation anywhere below the subderivation.

As with the other rules, you should be sure you understand why this rule works. Suppose in a subderivation we have drawn the conclusions **Y** and **~Y** from the assumption **X**. This is (by the rules for conjunction) equivalent to deriving the contradiction **Y&~Y** from **X**. Now, **X** must be either true or false. If it is true, and we have drawn from it the conclusion that **Y&~Y**, we have a valid argument from a true premise to a false conclusion. But that can't happen—our rules for derivations won't let that happen. So **X** must have been false, in which case **~X** must be true and can be entered as a conclusion in the outer derivation. Finally, if the subderivation has used premises or conclusions of the outer derivation, we can reason in exactly the same way, but subject to the restriction that we consider only cases in which the original premises were true.

In annotating negation introduction, keep in mind the same consideration which applied in annotating conditional introduction. The new line is justified by appeal, not to any one or two lines, but to a whole argument, represented by a subderivation. Consequently, the justification for the new line appeals to the whole subderivation. Indicate this fact by writing down the inclusive line numbers of the subderivation (the first and last of its line numbers separated by a dash).

In applying these rules, be sure to keep the following in mind: To apply the rules for conditional and negation introduction, you must always have a completed subderivation of the form shown. It's the presence of the subderivation of the right form which licenses the introduction of a conditional or a negated sentence. To apply any of the other rules, you must have the input sentence or sentences (the sentence or sentences in the box in the rule's schematic statement) to be licensed to write the output sentence of the rule (the sentence in the circle in the schematic presentation). But an input sentence can itself be either a prior conclusion in the derivation or an original premise or assumption.

Incidentally, you might have been puzzled by the rule for negation introduction. The rule for negation elimination has the form "~~**X.** Therefore **X**". Why not, you might wonder, use the rule "**X.** Therefore ~~**X**" for negation introduction? That's a good question. The rule "**X.** Therefore ~~**X**" is a correct rule in the sense that it is truth preserving. It will never get you a false sentence out of true ones. But the rule is not strong enough. For example, given the other rules, if you restrict yourself to the rule "**X.** Therefore ~~**X**" for negation introduction, you will never be able to construct a derivation that shows the argument

~A
———
~(A&B)

to be valid. We want our system of natural deduction not only to be *Sound*, which means that every derivation represents a valid argument. We also want it to be *Complete*, which means that every valid argument is represented by a derivation. If we use the rule "**X.** Therefore ~~**X**" for negation introduction, our system of natural deduction will not be complete. The rules will not be strong enough to provide a correct derivation for every valid argument.

EXERCISES

5–3. Below you find some correct derivations without the annotations which tell you, for each line, which rule was used in writing the line and to which previous line or lines the rule appeals. Copy the derivations and add the annotations. That is, for each line, write the line number of the previous line or lines and the rule which, applying to those previous lines, justifies the line you are annotating.

a)
```
1 | B&(B⊃~A)   P
  |----------
2 | B
3 | B⊃~A
4 | ~A
```

b)
```
1 | ~C≡(AvB)   P
2 | A          P
  |----------
3 | (AvB)⊃~C
4 | AvB
5 | ~C
```

c)
```
1 | A⊃~B       P
2 | BvC        P
  |----
3 |    | A
  |    |------
4 |    | A⊃~B
5 |    | ~B
6 |    | BvC
7 |    | C
8 | A⊃C
```


d)
```
1 | D          P
2 | (D&A)⊃C    P
  |----------
3 |    | A
  |    |----
4 |    | D
5 |    | D&A
6 |    | (D&A)⊃C
7 |    | C
8 | A⊃C
```

e)
```
1 | AvB        P
  |----
2 |    | ~A&~B
  |    |------
3 |    | ~A
4 |    | ~B
5 |    | AvB
6 |    | B
7 | ~(~A&~B)
```

f)
```
1  | A&B       P
   |----
2  |    | A
   |    |----
3  |    | A&B
4  |    | B
5  | A⊃B
6  |    | B
   |    |----
7  |    | A&B
8  |    | A
9  | B⊃A
10 | A≡B
```

g)
```
1 | ~A⊃B       P
2 | ~A⊃~B      P
  |----------
3 |    | ~A
  |    |------
4 |    | ~A⊃B
5 |    | B
6 |    | ~A⊃~B
7 |    | ~B
8 | ~~A
9 | A
```

5–4. For each of the following arguments, provide a derivation which shows the argument to be valid. Follow the same directions as you did for exercises 5–1 and 5–2.

a) C&~H / ~H

b) JvD, ~~~D / J

c) A&B / B&A

d) A⊃~D, ~~A / ~D

e) G⊃D, G⊃~D / ~G

f) A≡~B / ~B⊃A

g) M, Rv~H / M&(Rv~H)

h) A&(B&C) / (A&B)&C

i) ~C⊃D, D⊃~C / D≡~C

j) A≡~B, ~B / A

k) ~C⊃~~~A, ~C / ~A

l) K⊃~B, B&F / ~K

m) ~P, ~Q / ~(PvQ)

n) (N⊃K)&(N⊃L) / N⊃(K&L)

o) $D \supset (A \vee F)$
$D \supset \sim F$
$\sim A$

$\sim D \& \sim A$

p) $H \equiv J$
$H \equiv K$

$J \equiv K$

5–5. In chapter 3 we defined triple conjunctions and disjunctions, that is, sentences of the form **X&Y&Z** and **XvYvZ.** Write introduction and elimination rules for such triple conjunctions and disjunctions.

5–6. Suppose we have a valid argument and an assignment of truth values to sentence letters which makes one or more of the premises **false.** What, then, can we say about the truth value of the conclusions which follow validly from the premises? Do they have to be false? Can they be true? Prove what you say by providing illustrations of your answers.

CHAPTER SUMMARY EXERCISES

Give brief explanations for each of the following, referring back to the text to make sure your explanations are correct and saving your answers in your notebook for reference and review.

a) Derivation
b) Subderivation
c) Outer Derivation
d) Scope Line
e) Premise
f) Assumption
g) Rule of Inference
h) License (to draw a conclusion)
i) Truth Preserving Rule
j) Discharging an Assumption
k) Conjunction Introduction
l) Conjunction Elimination
m) Disjunction Introduction
n) Disjunction Elimination
o) Conditional Introduction
p) Conditional Elimination
q) Biconditional Introduction
r) Biconditional Elimination
s) Negation Introduction
t) Negation Elimination
u) Reiteration