Transcription 4

4-1. RESTRICTED QUANTIFIERS

For three chapters now I have been merrily transcribing '(∃x)' both as 'something' and 'someone', and I have been transcribing '(∀x)' both as 'everything' and 'everyone.' I justified this by saying that when we talked only about people we would restrict the variables 'x', 'y', etc. to refer only to people, and when we talked about everything, we would let the variables be unrestricted. It is actually very easy to make precise this idea of restricting the universe of discourse. If we want the universe of discourse to be restricted to people, we simply declare that all the objects in our interpretations must be people. If we want a universe of discourse consisting only of cats, we declare that all the objects in our interpretations must be cats. And so on.

As I mentioned, this common practice is not fully satisfactory. What if we want to talk about people and things, as when we assert, 'Everyone likes sweet things.'? Restricted quantifiers will help us out here. They also have the advantage of getting what we need explicitly stated in the predicate logic sentences themselves.

We could proceed by using '(∃x)' and '(∀x)' to mean 'something' and 'everything' and introduce new quantifiers for 'someone' and 'everyone'. To see how to do this, let's use the predicate 'P' to stand for 'is a person.'

Then we can introduce the new quantifier '$(\exists x)_P$' to stand for some x chosen from among the things that are P, that is, chosen from among people. We call this a restricted quantifier. You should think of a restricted quantifier as saying exactly what an unrestricted quantifier says except that the variable is restricted to the things of which the subscripted predicate is true. With 'P' standing for 'is a person', '$(\exists x)_P$' has the effect of 'someone' or 'somebody'. We can play the same game with the universal quantifier. '$(\forall x)_P$' will mean all x chosen from among the things that are P. With 'P' standing for 'is a person', ('$(\forall x)_P$' means, not absolutely everything, but all people, that is, everyone or everybody or anyone or anybody.

This notion of a restricted quantifier can be useful for other things. Suppose we want to transcribe 'somewhere' and 'everywhere' or 'sometimes' and 'always'. Let's use 'N' stand for 'is a place' or 'is a location'. 'Somewhere' means 'at some place' or 'at some location'. So we can transcribe 'somewhere' as '$(\exists x)_N$' and 'everywhere' as '$(\forall x)_N$'. For example, to transcribe 'There is water everywhere', I would introduce the predicate 'Wx' to stand for 'there is water at x'. Then '$(\forall x)_N Wx$' says that there is water everywhere. Continuing the same strategy, let's use 'Q' to stand for 'is a time'. Then '$(\exists x)_Q$' stands for 'sometime(s)' and '$(\forall x)_Q$' stands for 'always' ('at all times').

In fact, we can also use the same trick when English has no special word corresponding to the restricted quantifier. Suppose I want to say something about all cats, for example, that all cats are furry. Let 'Cx' stand for 'x is a cat' and 'Fx' stand for 'x is furry'. Then '$(\forall x)_C Fx$' says that all things which are cats are furry; that is, all cats are furry. Suppose I want to say that some animals have tails. Using 'Ax' for 'x is an animal' and 'Txy' for 'x is a tail of y', I write '$(\exists x)_A(\exists y)Tyx$': There is an animal, x, and there is a thing, y, such that y is a tail of x.

As you will see, restricted quantifiers are very useful in figuring out transcriptions, but there is a disadvantage in introducing them as a new kind of quantifier in our system of logic. If we have many different kinds of quantifiers, we will have to specify a new rule for each of them to tell us the conditions under which a sentence formed with the quantifier is true. And when we get to checking the validity of arguments, we will have to have a new rule of inference to deal with each new quantifier. We could state the resulting mass of new rules in a systematic way. But the whole business would still require a lot more work. Fortunately, we don't have to do any of that, for we can get the full effect of restricted quantifiers with the tools we already have.

Let's see how to rewrite subscripted quantifiers. Consider the restricted quantifier '$(\exists x)_C$', which says that there is cat such that, or there are cats such that, or some cats are such that. We say 'some cats are furry' (or 'there is a furry cat' or the like) with '$(\exists x)_C Fx$'. Now what has to be true for it to be true that some cats are furry, or that there is a furry cat? There has to be one or more things that is both a cat and is furry. If there is not something which is both a cat and is furry, it is false that there is a furry cat. So we can say that some cats are furry by writing '(∃x)(Cx & Fx)'. In short, we can faithfully rewrite '$(\exists x)_C$Fx' as '(∃x)(Cx & Fx)'. This strategy will work generally:

> Rule for rewriting *Subscripted Existential Quantifiers:* For any predicate **S**, any sentence of the form $(\exists \mathbf{u})_{\mathbf{S}}$(. . . **u**. . .) is shorthand for (∃**u**)[**Su** & (. . . **u** . . .)].

Here are some examples:

Some cats are blond.	$(\exists x)_C$Bx	(∃x)(Cx & Bx)
Eve loves a cat.	$(\exists x)_C$Lex	(∃x)(Cx & Lex)
Eve loves a furry cat.	$(\exists x)_C$(Fx & Lex)	(∃x)[Cx & (Fx & Lex)]

Clearly, we can proceed in the same way with 'someone' and 'somebody':

Someone loves Eve.	$(\exists x)_P$Lxe	(∃x)(Px & Lxe)
Somebody loves Eve or Adam.	$(\exists x)_P$(Lxe v Lxa)	(∃x)[Px &(Lxe v Lxa)]
If somebody loves Eve, then Eve loves somebody.		
	$(\exists x)_P$Lxe ⊃ $(\exists x)_P$Lex	(∃x)(Px & Lxe) ⊃ (∃x)(Px & Lex)

Notice that in the last example I used the rule for rewriting the subscript on each of two sentences **X** and **Y, inside** a compound sentence, **X ⊃ Y.**

How should we proceed with restricted universal quantifiers? This is a little tricky. Let's work on '$(\forall x)_C$Fx'—that is, 'All cats are furry'. Under what conditions is this sentence true? To answer the question, imagine that everything in the world is lined up in front of you: All the cats, dogs, people, stones, basketballs, everything. You go down the line and examine the items, one by one, to determine whether all cats are furry. If the first thing in line is a dog, you don't have to determine whether or not it is furry. If the second thing is a basketball, you don't have to worry about it either. But as soon as you come to a cat you must examine it further to find out if it is furry. When you finally come to the end of the line, you will have established that all cats are furry if you have found of each thing that, if it is a cat, then it is furry. In short, to say that all cats are furry is to say '(∀x)(Cx ⊃ Fx)'.

At this point, many students balk. Why, they want to know, should we rewrite a restricted universal quantifier with the '⊃' when we rewrite a restricted existential quantifier with the '&'? Shouldn't '&' work also for restricted universal quantifiers? Well, I'm sorry. It doesn't. That is just not what restricted universal quantifiers mean.

You can prove this for yourself by trying to use '&' in rewriting the subscripted 'C' in our transcription of 'All cats are furry.' You get

(1) (∀x)(Cx & Fx)

What does (1) say? It says that everything is a furry cat, and in particular that everything is a cat! That's much too strong. All cats could be furry even though there are lots of things which are not cats. Thus 'All cats are furry' could be true even when (1) is false, so that (1) cannot be the right way to rewrite '$(\forall x)_C$Fx'.

What has gone wrong? The unrestricted universal quantifier applies to everything. So we can use conjunction in expressing the restriction of cats only if we somehow disallow or except the cases of noncats. We can do this by saying that everything is either not a cat or is a cat and is furry:

(2) (∀x)[~Cx v (Cx & Fx)]

(2) does indeed say what 'All cats are furry' says. So (2) should satisfy your feeling that an '&' also comes into the restricted universal quantifier in some way. But you can easily show that (2) is logically equivalent to '(∀x)(Cx ⊃ Fx)'! As the formulation with the '⊃' is more compact, and is also traditional, it is the one we will use.

In general, we rewrite restricted universal quantifiers according to the rule

> Rule for rewriting *Subscripted Universal Quantifiers:* For any predicate **S**, any sentence of the form $(\forall \mathbf{u})_{\mathbf{S}}$(. . . **u** . . .) is shorthand for (∀**u**)[**Su** ⊃ (. . . **u** . . .)].

Here are some examples to make sure you see how this rule applies:

Eve loves all cats.	$(\forall x)_C$(Lex)	(∀x)(Cx ⊃ Lex)
Everybody loves Eve.	$(\forall x)_P$Lxe	(∀x)(Px ⊃ Lxe)
Everyone loves either Adam or Eve.		
	$(\forall x)_P$(Lxa v Lxe)	(∀x)[Px ⊃ (Lxa v Lxe)]
Not everyone loves both Adam and Eve.		
	~$(\forall x)_P$(Lxa & Lxe)	~(∀x)[Px ⊃ (Lxa & Lxe)]

In the last example, I used the rewriting rule on a sentence, **X, inside** a negated sentence of the form **~X.**

If you are still feeling doubtful about using the '⊃' to rewrite restricted universal quantifiers, I have yet another way to show you that this way of rewriting must be right. I am assuming that you agree that our way of rewriting restricted existential quantifiers is right. And I will use a new rule of logical equivalence. This rule tells us that the same equivalences that hold for negated unrestricted universal quantifiers hold for negated restricted universal quantifiers. In particular, saying that not all cats are furry is clearly the same as saying that some cat is not furry. In general

Rule ~$\forall_s$: A sentence of the form ~$(\forall u)_s$(. . . u . . .) is logically equivalent to $(\exists u)_s$~(. . . u . . .).

You can prove this new rule along the same lines we used in proving the rule ~∀.

Now, watch the following chain of logical equivalents:

1 $(\forall u)_s$(. . . u . . .)	
2 ~~$(\forall u)_s$(. . . u . . .)	DN
3 ~$(\exists u)_s$~(. . . u . . .)	~$\forall_s$
4 ~(∃u)[Su & ~(. . . u . . .)]	Rule for rewriting $(\exists u)_s$
5 ~(∃u)~~[Su & ~(. . . u . . .)]	DN
6 ~(∃u)~[~Su v (. . . u . . .)]	DM, DN
7 ~(∃u)~[Su ⊃ (. . . u . . .)]	C
8 ~~(∀u)[Su ⊃ (. . . u . . .)]	~∃
9 (∀u)[Su ⊃ (. . . u . . .)]	DN

Since the last line is logically equivalent to the first, it must be a correct way of rewriting the first.

If you are having a hard time following this chain of equivalents, let me explain the strategy. Starting with a restricted universal quantifier, I turn it into a restricted existential quantifier in lines 2 and 3 by using double denial and pushing one of the two negation signs through the restricted quantifier. I then get line 4 by using the rule we have agreed on for rewriting restricted existential quantifiers. Notice that I am applying this rule inside a negated sentence, so that here (and below) I am really using substitution of logical equivalents. In lines 5, 6, and 7 I use rules of logical equivalence to transform a conjunction into a conditional. These steps are pure sentence logic. They involve no quantifiers. Line 8 comes from line 7 by pushing the negation sign back out through what is now an unrestricted existential quantifier, changing it into an unrestricted universal quantifier. Finally, in line 9, I drop the double negation. It's almost like magic!

EXERCISES

4–1. Give an argument which shows that the rule ~$\forall_s$ is correct. Similarly, show that

Rule ~$\exists_x$: a sentence of the form ~$(\exists u)_s$(. . . u . . .) is logically equivalent to $(\forall u)_s$~(. . . u . . .).

is also correct.

4–2. Use the rule ~$\exists_s$ to show that, starting from the rule for rewriting subscripted universal quantifiers, you can derive the rule for rewriting subscripted existential quantifiers. Your argument will closely follow the one given in the text for arguing the rule for rewriting subscripted universal quantifiers from the rule for rewriting subscripted existential quantifiers.

4–3. Transcribe the following English sentences into the language of predicate logic. Use this procedure: In a first step, transcribe into a sentence using one or more subscripted quantifiers. Then rewrite the resulting sentence using the rules for rewriting subscripted quantifiers. Show both your first and second steps. Here are two examples of sentences to transcribe and the two sentences to present in presenting the problem:

Someone loves Eve.	All cats love Eve.
$(\exists x)_P$Lxe	$(\forall x)_C$Lxe
(∃x)(Px & Lxe)	(∀x)(Cx ⊃ Lxe)

Transcription Guide

e:	Eve	Dx:	x is a dog
Px:	x is a person	Bx:	x is blond
Cx:	x is a cat	Lxy:	x loves y

a) Everyone loves Eve.
b) Eve loves somebody.
c) Eve loves everyone.
d) Some cat loves some dog.
e) Somebody is neither a cat nor a dog.
f) Someone blond loves Eve.
g) Some cat is blond.
h) Somebody loves all cats.
i) No cat is a dog.
j) Someone loves someone.
k) Everybody loves everyone.
l) Someone loves everyone.
m) Someone is loved by everyone.
n) Everyone loves someone.
o) Everyone is loved by somebody.

4–2. TRANSCRIBING FROM ENGLISH INTO LOGIC

Transcribing into the language of predicate logic can be extremely difficult. Actually, one can do logic perfectly well without getting very good at transcription. But transcriptions into logic provide one of predicate logic's important uses. This is because, when it comes to quantification, English is often extremely confusing, ambiguous, and even downright obscure. Often we can become clearer about what is being said if we put a statement into logic. Sometimes transcribing into logic is a must for clarity and precision. For example, how do you understand the highly ambiguous sentence, 'All of the boys didn't kiss all of the girls.'? I, for one, am lost unless I transcribe into logic.

Before we get started, I should mention a general point. Just as in the case of sentence logic, if two predicate logic sentences are logically equivalent they are both equally good (or equally bad!) transcriptions of an English sentence. Two logically equivalent sentences share the same truth value in all possible cases (understood as all interpretations), and in this sense two logically equivalent sentences "say the same thing." But if two predicate logic sentences say the same thing, then to the extent that one of them says what an English sentence says, then so does the other.

We are going to be looking at quite a few examples, so let's agree on a transcription guide:

Transcription Guide

a:	Adam	Px:	x is a person
J:	The lights will be on	Rx:	x is a registered voter
Ax:	x is an adult	Vx:	x has the right to vote
Bx:	x is a boy	Kxy:	x kissed y
Cx:	x is a cat	Lxy:	x loves y
Dx:	x is a dog	Mxy:	x is married to y
Fx:	x can run a 3:45 mile	Oxy:	x owns y
Gx:	x is a girl	Txy:	x is a tail of y
Hx:	x is at home		

Take note of the fact that in giving you a transcription guide, I have been using open sentences to indicate predicates. For example, I am using the open sentence 'Px' to indicate the predicate 'is a person.' The idea of using an open sentence to indicate a predicate will soon become very useful.

To keep us focused on the new ideas, I will often use subscripts on restricted quantifiers. However, you should keep in mind that complete transcriptions require you to rewrite the subscripts, as explained in the last section.

Now let's go back and start with the basics. '$(\forall x)(Cx \supset Fx)$' transcribes 'all cats are furry,' 'Every cat is furry,' 'Any cat is furry,' and 'Each cat is furry.' This indicates that

> Usually, the words 'all', 'every', 'any', and 'each' signal a universal quantifier.

Let's make a similar list for the existential quantifier. '$(\exists x)(Cx \& Fx)$' transcribes 'Some cat is furry', 'Some cats are furry,' 'At least one cat is furry', 'There is a furry cat,' and 'There are furry cats':

> Usually, the expressions 'some', 'at least one', 'there is', and 'there are' signal an existential quantifier.

These lists make a good beginning, but you must use care. There are no hard and fast rules for transcribing English quantifier words into predicate logic. For starters, 'a' can easily function as a universal or an existential quantifier. For example, 'A car can go very fast.' is ambiguous. It can be used to say either that any car can go very fast or that some car can go very fast.

To make it clearer that 'a' can function both ways, consider the following examples. You probably understand 'A man is wise.' to mean that some man is wise. But most likely you understand 'A dog has four legs.' to mean that all dogs have four legs. Actually, both of these sentences are ambiguous. In both sentences, 'a' can correspond to 'all' or 'some'. You probably didn't notice that fact because when we hear an ambiguous sentence we tend to notice only one of the possible meanings. If a sentence is obviously true when understood with one of its meanings and obviously false when understood with the other, we usually hear the sentence only as making the true statement. So if all the men in the world were wise, we would take 'A man is wise.' to mean that all men are wise, and if only one dog in the world had four legs we would take 'A dog has four legs.' to mean that some dog has four legs.

It is a little easier to hear 'A car can go very fast.' either way. This is because we interpret this sentence one way or the other, depending on how fast we take 'fast' to be. If 'fast' means 30 miles an hour (which is very fast by horse and buggy standards), it is easy to hear 'A car can go very fast.' as meaning that all cars can go very fast. If "fast' means 180 miles an hour it is easy to hear 'a car can go very fast.' as meaning that some car can go very fast.

'A' is not the only treacherous English quantifier word. 'Anyone' usually gets transcribed with a universal quantifier. But not always. Consider

(3) If anyone is at home, the lights will be on.

(4) If anyone can run a 3:45 mile, Adam can.

We naturally hear (3), not as saying that if everyone is at home the lights will be on, but as saying that if **someone** is at home the lights will be on. So a correct transcription is

(3a) $(\exists x)_P Hx \supset J$

Likewise, by (4), we do not ordinarily mean that if everyone can run a 3:43 mile, Adam can. We mean that if **someone** can run that fast, Adam can:

(4a) $(\exists x)_P Fx \supset Fa$

At least that's what one would ordinarily mean by (4). However, I think that (4) actually is ambiguous. I think 'anyone' in (4) **could** be understood as 'everyone'. This becomes more plausible if you change the '3:45 mile' to '10-minute mile'. And it becomes still more plausible after you consider the following example: 'Anyone can tie their own shoe laces. And if anyone can, Adam can.'

Going back to (3), one would think that if (4) is ambiguous, (3) should be ambiguous in the same way. I just can't hear an ambiguity in (3). Can you?

'Someone' can play the reverse trick on us. Usually, we transcribe it with an existential quantifier. But consider

(5) Someone who is a registered voter has the right to vote.

We naturally hear this as the generalization stating that anyone who is a registered voter has the right to vote. Thus we transcribe it as

(5a) $(\forall x)_P(Rx \supset Vx)$

As in the case of (4), which uses 'anyone', we can have ambiguity in sentences such as (5), which uses 'someone'. If you don't believe me, imagine that you live in a totalitarian state, called Totalitarania. In Totalitarania, everyone is a registered voter. But voter registration is a sham. In fact, only one person, **the boss,** has the right to vote. As a citizen of Totalitarania, you can still truthfully say that someone who is a registered voter (namely, **the boss**) has the right to vote. (You can make this even clearer by emphasizing the word 'someone: '**someone** who is a registered voter has the right to vote.') In this context we hear the sentence as saying

(5b) $(\exists x)_P(Rx \& Vx)$

Ambiguity can plague transcription in all sorts of ways. Consider an example traditional among linguists:

(6) All the boys kissed all the girls

This can easily mean that each and every one of the boys kissed each and every one of the girls:

(6a) $(\forall x)_B(\forall y)_G Kxy$

But it can also mean that each of the boys kissed some girls so that, finally, each and every girl got kissed by some boy:

(6b) $(\forall x)_B(\exists y)_G Kxy \& (\forall y)_G(\exists x)_B Kxy$

If you think that was bad, things get much worse when multiple quantifiers get tangled up with negations. Consider

(7) All the boys didn't kiss all the girls.

Everytime I try to think this one through, I blow a circuit. Perhaps the most natural transcription is to take the logical form of the English at face value and take the sentence to assert that of each and every boy it is true that he did not kiss all the girls; that is, for each and every boy there is at least one girl not kissed by that boy:

(7a) $(\forall x)_B{\sim}(\forall y)_G Kxy$, or $(\forall x)_B(\exists y)_G{\sim}Kxy$

But one can also take the sentence to mean that each and every boy refrained from kissing each and every girl, that is, didn't kiss the first girl and didn't kiss the second girl and not the third, and so on. In yet other words, this says that for each and every boy there was no girl whom he kissed, so that nobody kissed anybody:

(7b) $(\forall x)_B(\forall y)_G{\sim}Kxy$, or $(\forall x)_B{\sim}(\exists y)_G Kxy$, or ${\sim}(\exists x)_B(\exists y)_G Kxy$

We are still not done with this example, for one can **also** use (7) to mean that not all the boys kissed every single girl—that is, that some boy did not kiss all the girls, in other words that at least one of the boys didn't kiss at least one of the girls:

(7c) ${\sim}(\forall x)_B(\forall y)_G Kxy$, or $(\exists x)_B{\sim}(\forall y)_G Kxy$, or $(\exists x)_B(\exists y)_G{\sim}Kxy$

It's worth an aside to indicate how it can happen that an innocent-looking sentence such as (7) can turn out to be so horribly ambiguous. Modern linguistics postulates that our minds carry around more than one representation of a given sentence. There is one kind of structure that represents the logical form of a sentence. Another kind of structure represents sentences as we speak and write them. Our minds connect these (and other) representations of a given sentence by making all sorts of complicated transformations. These transformations can turn representations of **different** logical forms into the **same** representation of a spoken or written sentence. Thus one sentence which you speak or write can correspond to two, three, or sometimes quite a few different structures that carry very different meanings. In particular, the written sentence (7) corresponds to (at least!) three different logical forms. (7a), (7b), and (7c) don't give all the details of the different, hidden structures that can be transformed into (7). But they do describe the differences which show up in the language of predicate logic.

You can see hints of all this if you look closely at (7), (7a), (7b), and (7c). In (7) we have two universal quantifier words and a negation. But since the quantifier words appear on either side of 'kissed', it's really not all that clear where the negation is meant to go in relation to the universal quantifiers. We must consider three possibilities. We could have the negation between the two universal quantifiers. Indeed, that is what you see in (7a), in the first of its logically equivalent forms. Or we could have the negation coming after the two universal quantifiers, which is what you find in the first of the logically equivalent sentences in (7b). Finally, we could have the negation preceding both universal quantifiers. You see this option in (7c). In sum, we have three similar, but importantly different, structures. Their logical forms all have two universal quantifiers and a negation, but the three differ, with the negation coming before, between, or after the two quantifiers. The linguistic transformations in our minds connect all three of these structures with the same, highly ambiguous English sentence, (7).

Let's get back to logic and consider some other words which you may find especially difficult to transcribe. I am always getting mixed up by sentences which use 'only', such as 'Only cats are furry.' So I use the strategy of first transcribing a clear case (it helps to use a sentence I know is true) and then using the clear case to figure out a formula. I proceed in this way: Transcribe

(8) Only adults can vote.

This means that anyone who is not an adult can't vote, or equivalently (using the law of contraposition), anyone who can vote is an adult. So either of the following equivalent sentences provides a correct transcription:

(8a) $(\forall x)_P(\sim Ax \supset \sim Vx)$

(8b) $(\forall x)_P(Vx \supset Ax)$

This works in general. (In the following I used boldface capital **P** and **Q** to stand for arbitrary predicates.) Transcribe

(9) Only **P**s are **Q**s.

either as

(9a) $(\forall x)(\sim \mathbf{P}x \supset \sim \mathbf{Q}x)$

or as

(9b) $(\forall x)(\mathbf{Q}x \supset \mathbf{P}X)$

Thus 'Only cats are furry' becomes $(\forall x)(Fx \supset Cx)$.

'Nothing' and 'not everything' often confuse me also. We must carefully distinguish

(10) Nothing is furry: $(\forall x)\sim Fx$, or $\sim(\exists x)Fx$

and

(11) Not everything is furry: $\sim(\forall x)Fx$, or $(\exists x)\sim Fx$

(The alternative transcriptions given in (10) and (11) are logically equivalent, by the rules $\sim(\forall x)$ and $\sim(\exists x)$ for logical equivalence introduced in section 3–4.) 'Not everything' can be transcribed literally as 'not all x . . .'. 'Nothing' means something different and much stronger. 'Nothing' means 'everything is not . . . .' Be careful not to confuse 'nothing' with 'not everything.' If the distinction is not yet clear, make up some more examples and carefully think them through.

'None' and 'none but' can also cause confusion:

(12) None but adults can vote: $(\forall x)(\sim Ax \supset \sim Vx)$

(13) None love Adam: $(\forall x)\sim Lxe$

'None but' simply transcribes as 'only.' When 'none' without the 'but' fits in grammatically in English you will usually be able to treat it as you do 'nothing'. 'Nothing' and 'none' differ in that we tend to use 'none' when there has been a stated or implied restriction of domain: "How many cats does Adam love? He loves none." In this context a really faithful transcription of the sentence 'Adam loves none.' would be '$(\forall x)_C\sim Lax$', or, rewriting the subscript, '$(\forall x)(Cx \supset \sim Lax)$.

Perhaps the most important negative quantifier expression in English is 'no', as in

(14) No cats are furry.

To say that no cats are furry is to say that absolutely all cats are not furry, so that we transcribe (18) as

(15) $(\forall x)_C\sim Fx$, that is, $(\forall x)(Cx \supset \sim Fx)$

In general, transcribe

(16) No **P**s are **Q**x.

as

(17) $(\forall x)_P{\sim}Q$, that is, $(\forall x)(P \supset {\sim}Q)$

EXERCISES

4–4. Transcribe the following English sentences into the language of predicate logic. Use subscripts if you find them helpful in figuring out your answers, but no subscripts should appear in your final answers.

Transcription Guide

a:	Adam	Fx:	x is furry
e:	Eve	Px:	x is a person
Ax:	x is an animal	Qx:	x purrs
Bx:	x is blond	Lxy:	x loves y
Cx:	x is a cat	Sxy:	x is a son of y
Dx:	x is a dog	Txy:	x is tickling y

a) Anything furry loves Eve.
b) No cat is furry.
c) If anyone loves Adam, Eve does.
d) Eve does not love anyone.
e) Nothing is furry.
f) Adam, if anyone, is blond.
g) Not all cats are furry.
h) Some cats are not furry.
i) No one is a cat.
j) No cat is a dog.
k) If something purrs, it is a cat.
l) Not everything blond is a cat.
m) A dog is not an animal. (Ambiguous)
n) Not all animals are dogs.
o) Only cats purr.
p) Not only cats are furry.
q) Any dog is not a cat.
r) No blonds love Adam.
s) None but blonds love Adam.
t) Some dog is not a cat.
u) Nothing furry loves anyone.
v) Only cat lovers love dogs. (Ambiguous?)
w) If someone is a son of Adam, he is blond.
x) No son of Adam is a son of Eve.

y) Someone who is a son of Adam is no son of Eve. (Ambiguous)
z) Each cat which loves Adam also loves Eve.
aa) Not everyone who loves Adam also loves Eve.
bb) Anyone who is tickling Eve is tickling Adam.
cc) None but those who love Adam also love Eve.

4–5. Give alternative transcriptions which show the ways in which the following sentences are ambiguous. In this problem you do not have to eliminate subscripts. (It is sometimes easier to study the ambiguity if we write these sentences in the compact subscript notation.)

a) Everyone loves someone.
b) Someone loves everyone.
c) Something is a cat if and only if Adam loves it.
d) All cats are not furry.
e) Not anyone loves Adam.

4–6. In this section I discussed ambiguities connected with words such as 'a', 'someone', and 'anyone.' In fact, English has a great many sorts of ambiguity arising from the ways in which words are connected with each other. For example, 'I won't stay at home to please you.' can mean that if I stay at home, I won't do it in order to please you. But it can also mean that I will go out because going out will please you. 'Eve asked Adam to stay in the house.' can mean that Eve asked Adam to remain in a certain location, and that location is the house. It can also mean that Eve asked Adam to remain in some unspecified location, and that she made her request in the house.

For the following English sentences, provide alternative transcripts showing how the sentences are ambiguous. Use the transcription guides given for each sentence.

a) Flying planes can be dangerous. (Px: x is a plane. Fx: x is flying. Dx: x can be dangerous. Ax: x is an act of flying a plane.)

b) All wild animal keepers are blond. (Kxy: x keeps y. Wx: x is wild. Ax: x is an animal. Bx: x is blond.)

c) Adam only relaxes on Sundays. (a: Adam. Rxy: x relaxes on day y. Lxy: x relaxes ("is lazy") all day long on day y. Sx: x is Sunday.)

d) Eve dressed and walked all the dogs. (e: Eve. Cxy: x dressed y. Dx: x is a dog. Wxy: x walked y.)

Linguists use the expression *Structural Ambiguity* for the kind of ambiguity in these examples. This is because the ambiguities have to do with alternative ways in which the grammatical structure of the sentences can be correctly analyzed. Structural ambiguity contrasts with *Lexical Ambiguity*, which has to do with the ambiguity in the meaning of isolated words. Thus the most obvious ambiguity of 'I took my brother's picture yesterday.' turns on the ambiguity of the meaning of 'took' (stole vs. produced a picture). The ambiguity involved with quantifier words such as 'a', 'someone', and 'anyone' is actually structural ambiguity, not lexical ambiguity. We can see a hint of this in the fact that '(∃x)Hx ⊃ J' is logically equivalent to '(∀x)(Hx ⊃ J) and the fact that '(∀x)Hx ⊃ J' is logically equivalent to (∃x)(Hx ⊃ J), as you will prove later on in the course.

4–3. TRANSCRIPTION STRATEGIES

I'm going to turn now from particularly hard cases to general strategy. If you are transcribing anything but the shortest of sentences, don't try to do it all at once. Transcribe parts into logic, writing down things which are part logic and part English. Bit by bit, transcribe the parts still in English into logic until all of the English is gone.

Let's do an example. Suppose we want to transcribe

(18) Any boy who loves Eve is not a furry cat.

(18) says of any boy who loves Eve that he is not a furry cat; that is, it says of all things, x, of which a first thing is true (that x is a boy who loves Eve) that a second thing is true (x is not a furry cat). So the sentence has the form (∀x)(**P**x ⊃ **Q**):

(18a) (∀x)(x is a boy who loves Eve ⊃ x is not a furry cat)

Now all you have to do is to fashion transcriptions of 'x is a boy who loves Eve' and of 'x is not a furry cat' and plug them into (18a):

(18b) x is a boy who loves Eve: Bx & Lxe

(18c) x is not a furry cat: ~(Fx & Cx)

(Something which is not a furry cat is not both furry and a cat. Such a thing could be furry, or a cat, but not both.) Now we plug (18b) and (18c) into (18a), getting our final answer:

(18d) (∀x)[(Bx & Lxe) ⊃ ~(∃x & Cx)]

Here is another way you could go about the same problem. Think of the open sentence 'Bx & Lxe' as indicating a complex one place predicate. The open sentence 'Bx & Lxe' presents something which might be true

of an object or person such as Adam. For example, if the complex predicate is true of Adam, we would express that fact by writing in 'a' for 'x' in 'Bx & Lxe', giving 'Ba & Lae'. Now, thinking of 'Bx & Lxe' as a predicate, we can use the method of quantifier subscripts which we discussed in section 4–1. (18) is somewhat like a sentence which asserts that everything is not a furry cat. But (18) asserts this, not about absolutely everything, but just about all those things which have the complex property Bx & Lxe. So we can write (18) as a universally quantified sentence with the universal quantifier restricted by the predicate 'Bx & Lxe':

(18e) $(\forall x)_{(Bx\ \&\ Lxe)}$~(Fx & Cx)

Now you simply use the rule for rewriting subscripts on universal quantifiers, giving (18d).

In yet a third way of working on (18), you could first use the method of subscripting quantifiers before transcribing the complex predicates into logic. Following this route you would first write.

(18f) $(\forall x)_{(x\text{ is a boy who loves Eve})}$(x is not a furry cat)

Now transcribe the English complex predicates as in (18b) and (18c), plug the results into (18f), giving (18e). Then you rewrite the subscript, giving (18d) as before. You have many alternative ways of proceeding.

Generally, it is very useful to think of complex descriptions as complex predicates. In particular, this enables us to use two place predicates to construct one place predicates. We really took advantage of this technique in the last example. 'Lxy' is a two place predicate. By substituting a name for 'y', we form a one place predicate, for example, 'Lxe'. 'Lxe' is a one place predicate which is true of anything which loves Eve.

Here is another useful way of constructing one place predicates from two place predicates. Suppose we need the one place predicate 'is married', but our transcription guide only gives us the two place predicate 'Mxy', meaning that x is married to y. To see how to proceed, consider what it means to say that Adam, for example, is married. This is to say that there is someone to whom Adam is married. So we can say Adam is married with the sentence '(∃y)May'. We could proceed in the same way to say that Eve, or anyone else, is married. In short, the open sentence '(∃y)Mxy' expresses the predicate 'x is married'.

Here's another strategy point: When 'who' or 'which' comes after a predicate they generally transcribe as 'and'. As you saw in (18), the complex predicate 'x is a boy who loves Eve' becomes 'Bx & Lxe'. The complex predicate 'x is a dog which is not furry but has a tail' becomes 'Dx & (~Fx & (∃y)Tyx)'.

When 'who' or 'which' comes after a quantifier word, they indicate a subscript on the quantifier: 'Anything which is not furry but has a tail' should be rendered as $(\forall x)_{(\sim Fx\ \&\ (\exists y)Tyx)}$. When the quantifier word itself calls for a subscript, as does 'someone', you need to combine both these ideas for treating 'who': 'Someone who loves Eve' is the subscripted quantifier '$(\exists x)_{Px \,\&\, Lxe}$'.

Let's apply these ideas in another example. Before reading on, see if you can use only 'Cx' for 'x is a cat', 'Lxy' for 'x loves y', and 'Oxy' for 'x owns y' and transcribe

(19) Some cat owner loves everyone who loves themselves.

Let's see how you did. (19) says that there is something, taken from among the cat owners, and that thing loves everyone who loves themselves. Using a subscript and the predicates 'x is a cat owner' and 'x loves everyone who loves themselves', (19) becomes

(19a) $(\exists x)_{(\text{x is a cat owner})}$(x loves everyone who loves themselves)

Now we have to fashion transcriptions for the two complex English predicates used in (19a). Someone (or something) is a cat owner just in case there is a cat which they own:

(19b) x is a cat owner: $(\exists y)(Cy \,\&\, Oxy)$

To say that x loves everyone who loves themselves is to say that x loves, not absolutely everyone, but everyone taken from among those that are, first of all people, and second, things which love themselves. So we want to say that x loves all y, where y is restricted to be a person, Py, and restricted to be a self-lover, Lyy:

(19c) x loves everyone who loves themselves: $(\forall y)_{(Py \,\&\, Lyy)}Lxy$

Putting the results of (19b) and (19c) into (19a), we get

(19d) $(\exists x)_{(\exists y)(Cy \,\&\, Oxy)}[(\forall y)_{(Py \,\&\, Lyy)}Lxy]$

Discharging first the subscript of '$(\exists x)$' with an '&' and then the subscript of '$(\forall y)$' with a '$\supset$', we get

(19e) $(\exists x)\{(\exists y)(Cy \,\&\, Oxy) \,\&\, (\forall y)_{(Py \,\&\, Lyy)}Lxy\}$

(19f) $(\exists x)\{(\exists y)(Cy \,\&\, Oxy) \,\&\, (\forall y)[(Py \,\&\, Lyy) \supset Lxy]\}$

This looks like a lot of work, but as you practice, you will find that you can do more and more of this in your head and it will start to go quite quickly.

I'm going to give you one more piece of advice on transcribing. Suppose you start with an English sentence and you have tried to transcribe it into logic. In many cases you can catch mistakes by transcribing your

logic sentence back into English and comparing your retranscription with the original sentence. This check works best if you are fairly literal minded in retranscribing. Often the original and the retranscribed English sentences will be worded differently. But look to see if they still seem to say the same thing. If not, you have almost certainly made a mistake in transcribing from English into logic.

Here is an illustration. Suppose I have transcribed

(20) If something is a cat, it is not a dog.

as

(20a) $(\exists x)(Cx \supset \sim Dx)$

To check, I transcribe back into English, getting

(20b) There is something such that if it is a cat, then it is not a dog.

Now compare (20b) with (20). To make (20b) true it is enough for there to be **one** thing which, if a cat, is not a dog. The truth of (20b) is consistent with there being a million cat-dogs. But (20) is not consistent with there being any cat-dogs. I conclude that (20a) is a wrong transcription. Having seen that (20) is stronger than (20a), I try

(20c) $(\forall x)(Cx \supset \sim Dx)$

Transcribing back into English this time gives me

(20d) Everything which is a cat is not a dog.

which does indeed seem to say what (20) says. This time I am confident that I have transcribed correctly.

(Is (20) ambiguous in the same way that (5) was? I don't think so!)

Here is another example. Suppose after some work I transcribe

(21) Cats and dogs have tails.

as

(21a) $(\forall x)[(Cx \,\&\, Dx) \supset (\exists y)Txy]$

To check, I transcribe back into English:

(21b) Everything is such that if it is both a cat and a dog, then it has a tail.

Obviously, something has gone wrong, for nothing is both a cat and a dog. Clearly, (21) is not supposed to be a generalization about such imaginary cat-dogs. Having noticed this, I see that (21) is saying one thing about cats and then the **same** thing about dogs. Thus, without further work, I try the transcription

(21c) $(\forall x)(Cx \supset (\exists y)Txy) \& (\forall x)(Dx \supset (\exists y)Txy)$

To check (21c), I again transcribe back into English, getting

(21d) If something is a cat, then it has a tail, and if something is a dog, then it has a tail.

which is just a long-winded way of saying that all cats and dogs have tails—in other words, (21). With this check, I can be very confident that (21c) is a correct transcription.

EXERCISES

Use this transcription guide for exercises 4–7 and 4–8:

a:	Adam	Fx:	x is furry
e:	Eve	Px:	x is a person
Ax:	x is an animal	Qx:	x purrs
Bx:	x is blond	Lxy:	x loves y
Cx:	x is a cat	Sxy:	x is a son of y
Dx:	x is a dog	Txy:	x is a tail of y
		Oxy:	x owns y

4–7. Transcribe the following sentences into English:

a) $(\exists x)(\exists y)(Px \& Py \& Sxy)$
b) $\sim(\exists x)(Px \& Ax)$
c) $\sim(\forall x)[Qx \supset (Fx \& Cx)]$
d) $(\exists x)[Qx \& \sim(Fx \& Cx)]$
e) $(\forall x)\sim[Px \& (Lxa \& Lxe)]$
f) $(\forall x)[Px \supset \sim(Lxa \& Lxe)]$
g) $(\forall x)(\forall y)[(Dx \& Cy) \supset Lxy]$
h) $(\forall x)(\forall y)[Dx \supset (Cy \supset Lxy)]$
i) $(\exists x)[Px \& (\exists y)(\exists z)(Py \& Szy \& Lxz)]$
j) $(\exists x)[Px \& (\exists y)(\exists z)(Pz \& Syz \& Lxz)]$
k) $(\forall x)\{[Bx \& (\exists y)(Fy \& Txy)] \supset (\exists z)(Cz \& Txz)\}$
l) $(\forall x)\{(\exists y)Sxy \supset [(\exists z)(Cz \& Lxz) \equiv (\exists z)(Dz \& Lxz)]\}$

4–8. Transcribe the following sentences into predicate logic. I have included some easy problems as a review of previous sections along with some real brain twisters. I have marked the sentences which

seem to me clearly ambiguous, and you should give different transcriptions for these showing the different ways of understanding the English. Do you think any of the sentences I haven't marked are also ambiguous? You should have fun arguing out your intuitions about ambiguous cases with your classmates and instructor.

a) All furry cats purr.
b) Any furry cat purrs.
c) No furry cats purr.
d) None of the furry cats purr.
e) None but the furry cats purr. (Ambiguous?)
f) Some furry cats purr.
g) Some furry cats do not purr.
h) Some cats and dogs love Adam.
i) Except for the furry ones, all cats purr.
j) Not all furry cats purr.
k) If a cat is furry, it purrs.
l) A furry cat purrs. (Ambiguous)
m) Only furry cats purr.
n) Adam is not a dog or a cat.
o) Someone is a son.
p) Some sons are blond.
q) Adam loves a blond cat, and Eve loves one too.
r) Adam loves a blond cat and so does Eve. (Ambiguous)
s) Eve does not love everyone.
t) Some but not all cats are furry.
u) Cats love neither Adam nor Eve.
v) Something furry loves Eve.
w) Only people love someone.
x) Some people have sons.
y) Any son of Adam is a son of Eve.
z) Adam is a son and everybody loves him.
aa) No animal is furry, but some have tails.
bb) Any furry animal has a tail.
cc) No one has a son.
dd) Not everyone has a son.
ee) Some blonds love Eve, some do not.
ff) Adam loves any furry cat.
gg) All blonds who love themselves love Eve.
hh) Eve loves someone who loves themself.
ii) Anyone who loves no cats loves no dogs.
jj) Cats love Eve if they love anyone. (Ambiguous)
kk) If anyone has a son, Eve loves Adam. (Ambiguous)
ll) If anyone has a son, that person loves Adam.

mm) Anyone who has a son loves Eve.
nn) If someone has a son, Adam loves Eve.
oo) If someone has a son, that person loves Adam.
pp) Someone who has a son loves Adam. (Ambiguous)
qq) All the cats with sons, except the furry ones, love Eve.
rr) Anyone who loves a cat loves an animal.
ss) Anyone who loves a person loves no animal.
tt) Adam has a son who is not furry.
uu) If Adam's son has a furry son, so does Adam.
vv) A son of Adam is a son of Eve. (Ambiguous)
ww) If the only people who love Eve are blond, then nobody loves Eve.
xx) No one loves anyone. (Ambiguous)
yy) No one loves someone. (Ambiguous)
zz) Everyone loves no one.
aaa) Everyone doesn't love everyone. (Ambiguous!)
bbb) Nobody loves nobody. (Ambiguous?)
ccc) Except for the furry ones, every animal loves Adam.
ddd) Everyone loves a lover. (Ambiguous)
eee) None but those blonds who love Adam own cats and dogs.
fff) No one who loves no son of Adam loves no son of Eve.
ggg) Only owners of dogs with tails own cats which do not love Adam.
hhh) None of Adam's sons are owners of furry animals with no tails.
iii) Anyone who loves nothing without a tail owns nothing which is loved by an animal.
jjj) Only those who love neither Adam nor Eve are sons of those who own none of the animals without tails.
kkk) Anyone who loves all who Eve loves loves someone who is loved by all who love Eve.

4–9. Transcribe the following sentences into predicate logic, making up your own transcription guide for each sentence. Be sure to show as much of the logical form as possible.

a) No one likes Professor Snarf.
b) Any dog can hear better than any person.
c) Neither all Republicans nor all Democrats are honest.
d) Some movie stars are better looking than others.
e) None of the students who read **A Modern Formal Logic Primer** failed the logic course.
f) Only people who eat carrots can see well in the dark.
g) Not only people who eat carrots can see as well as people who eat strawberries.
h) Peter likes all movies except for scary ones.

i) Some large members of the cat family can run faster than any horse.
j) Not all people with red hair are more temperamental than those with blond hair.
k) Some penny on the table was minted before any dime on the table.
i) No pickle tastes better than any strawberry.
m) John is not as tall as anyone on the basketball team.
n) None of the pumpkins at Smith's fruit stand are as large as any of those on MacGreggor's farm.
o) Professors who don't prepare their lectures confuse their students.
p) Professor Snarf either teaches Larry or teaches someone who teaches Larry.
q) Not only logic teachers teach students at Up State U.
r) Anyone who lives in Boston likes clams more than anyone who lives in Denver. (Ambiguous)
s) Except for garage mechanics who fix cars, no one has greasy pants.
t) Only movies shown on channel 32 are older than movies shown on channel 42.
u) No logic text explains logic as well as some professors do.
v) The people who eat, drink, and are merry are more fun than those who neither smile nor laugh.

CHAPTER SUMMARY EXERCISES

In reviewing this chapter make a short summary of the following to ensure your grasp of these ideas:

a) Restricted Quantifiers
b) Rule $\sim\forall_S$
c) Rule $\sim\exists_S$
d) Transcription Guide
e) Words that generally transcribe with a universal quantifier
f) Word that generally transcribe with an existential quantifier
g) Negative Quantifier Words
h) Ambiguity
i) Give a summary of important transcription strategies