CONTENTS

ANSWERS TO EXERCISES IN VOLUME I, CHAPTER 1

1-1. An argument is a bunch of sentences. One of them is called the conclusion and the rest are called the premises. The idea is that if you believe that the premises are true, then that should give you some reason for believing that the conclusion is true also.

There are a great many different examples you could give of both inductive and deductive argument. Here is are examples which are a little different from the ones given in the text:

Inductive:

High government spending causes interest rates to go up.
The government is spending a lot.
____
Interests rates will go up.

Deductive:

Either the government will not spend a lot or interest rates will go up.
The government will spend a lot.
____
Interest rates will go up.

In the inductive argument, if you really believe the premises, you are going to think that the conclusion has a good chance of being true. But even if there is the causal connection between government spending and interest rates claimed in the first premise, the connection is not sure fire. Many factors other than government spending influence interest rates, for example the demand for loans by businesses and home owners. In the case of the deductive argument, if the premises are true the conclusion has got to be true also. For suppose that the first premise is true. If the second premise is true also the first disjunct of the first premise is false. So the only way left for the first premise to be true is for its second disjunct to be true, which is the conclusion.

1-2. a) G&F, b) G&F, c) Gv-F, d) -G&-F, e) -(G&F), f) -G&-F, g) -(GvF), h) -Gv-F, i) -(GvF)

1-3. A non-truth-functional sentence is a longer sentence built up from one or more shorter component sentences in which the truth value of the whole is NOT determined JUST by the truth values of the components. In other words, in order to figure out the truth value of the whole you would have to know more than just the truth value of the component or components. Here are some examples: 'Smith believes that Tokyo has a larger population than New York.' The expression 'Smith believes that' can be regarded as a connective. Put it in front of a sentence and you get a new, longer sentence. But knowing the truth value of the shorter sentence is not enough to determine the truth value of the longer sentence. Knowing whether Tokyo has a larger population than New York will not tell you whether Smith believes that Tokyo has a larger population than New York. Here is another example. Suppose we are flipping a penny. Consider: 'The penny comes up heads is more probable than the penny comes up tails.' Knowing the truth value of 'The penny comes up heads.' and 'The penny comes up tails.' is not enough to know whether coming up heads is (or was) more likely than coming up tails. To know whether thi is true you need to know whether or not the coin is weighted to make heads more likely than tails.

1-4.

Sentence	Main Connective	Components
b) (D&-G)v(G&D)	v	D&-G, G&D
D&-G	&	D, -G
-G	-	G
G&D	&	G, D
c) [(Dv--B)&(DvB)]&(DvB)	&	(Dv--B)&(DvB), DvB
(Dv--B)&(DvB)	&	Dv--B, DvB
Dv--B	v	D, --B
--B	-	-B
-B	-	B
DvB	v	D, B
d) L&{Mv[-N&(Mv-L)]}	&	L, {Mv[-N&(Mv-L)]}
Mv[-N&(Mv-L)]}	v	M, -N&(Mv-L)
-N&(Mv-L)	&	-N, Mv-L
-N	-	N
Mv-L	v	M, -L
-L	-	L

1-5. b) is not a proper sentence logic sentence. There is no way to get tl expression by building it up by the formation rules from shorter sentence logic sentences. '-' applies only to a sentence, and no sentence begins wit '&'. f) is also not a sentence logic sentence. In this case the problem i that parentheses are missing which would tell us whether the sentence was supposed to be a conjunction of two shorter sentences or a disjunction of t shorter sentences.

The other expressions are all proper sentence logic sentences.

1-6.

A	B	-B	a) -BvA	BvA	b) -(BvA)
t	t	f	t	t	f
t	f	t	t	t	f
f	t	f	f	t	f
f	f	t	t	f	t

c)

Q	T	QvT	-Q	-T	-Qv-T	(QvT)&(-Qv-T)
t	t	t	f	f	f	f
t	f	t	f	t	t	t
f	t	t	t	f	t	t
f	f	f	t	t	t	f

d)

D	G	-G	D&-G	G&D	(D&-G)v(G&D)
t	t	f	f	t	t
t	f	t	t	f	t
f	t	f	f	f	f
f	f	f	f	f	f

e)

A	B	C	-B	-BvC	Av(-BvC)
t	t	t	f	t	t
t	t	f	f	f	t
t	f	t	t	t	t
t	f	f	t	t	t
f	t	t	f	t	t
f	t	f	f	f	f
f	f	t	t	t	t
f	f	f	t	t	t

f)

K	P	M	-P	-PvM	-P&(-PvM)	Kv[-P&(-PvM)]
t	t	t	f	t	f	t
t	t	f	f	f	f	t
t	f	t	t	t	t	t
t	f	f	t	t	t	t
f	t	t	f	t	f	f
f	t	f	f	f	f	f
f	f	t	t	t	t	t
f	f	f	t	t	t	t

g)

D	B	-B	--B	Dv--B	Dv-B	DvB	(Dv--B)&(Dv-B)	[(Dv--B)&(Dv-B)]&(DvB)
t	t	f	t	t	t	t	t	t
t	f	t	f	t	t	t	t	t
f	t	f	t	t	f	t	f	f
f	f	t	f	f	t	f	f	f

h)

L	M	N	-M	-N	-L	-Mv-L	-N&(-Mv-L)	Mv[-N&(-Mv-L)]	L&{Mv[-N&(-Mv-L)]}
t	t	t	f	f	f	f	f	t	t
t	t	f	f	t	f	f	f	t	t
t	f	t	t	f	f	t	f	f	f
t	f	f	t	t	f	t	t	t	t
f	t	t	f	f	t	t	f	t	f
f	t	f	f	t	t	t	t	t	f
f	f	t	t	f	t	t	f	f	f
f	f	f	t	t	t	t	t	t	f

1-7. Quotation marks function to form the NAME of a letter or expression. 'A' is a name of A. Thus, when I want to talk about a sentence letter, or a compound sentence, I put quotes around the expression. On the other hand, I use no quotes when I USE an expression. Also I am using bold face capitals to talk about sentences generally.

Philosophers refer to this distinction as the distinction between Use and Mention.

ANSWERS TO EXERCISES IN VOLUME I, CHAPTER 2

2-1. To say that Adam is not both ugly and dumb is to say that he is not ugly, or he is not dumb (or possibly not either ugly or dumb): '-Uv-D'. Equally, this comes to saying that he is not both ugly and dumb: -(U&D). If you think about these two expressions, I hope you will see that they come to the same thing. We shall prove that they are logically equivalent in the next chapter. But '-U&-D' is an incorrect transcription. Transcribe it back into English, and you get: 'Adam is not ugly and not dumb,' which is saying something stronger than that Adam is not both ugly and dumb.

2-2. a) CvD, b) CvD, c) C&D, d) C&-D, e) -Cv-D, f) -Cv-D, g) D&A, h) (B&A)v(-B&E), i) -D&(CvE), j) D&-A, k) (BvA)&-E, l) (DvB)&-A, m) (C&-E)&A, n) A&-D, o) A&E, p) (Cv-D)&(BvA), q) (E&-D)&(Bv-A), r) -A&[B&(CvE)], s) Av-A, t) (AvA)&[C&(DvE)], u) Bv[E&(DvC)], v) [(Bv-A)&(DvE)]v[(Av-B)&(C&-D)].

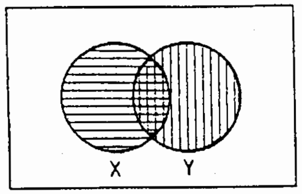
2-3. a) Either Adam is blond or not.
b) Adam loves Eve and is not blond.
c) Neither is Adam in love with Eve nor is Eve clever.
d) Adam is blond, or Eve is dark eyed and not clever.
e) Eve is either in love with Adam or not clever, and either Adam is not blond or he is in love with Eve.
f) Either it is the case that both Adam loves Eve or Eve loves Adam and Eve is not clever, or it is the case that Eve is clever and not dark eyed.
g) Eve is clever. Furthermore, either it is not both true that Eve loves Adam and Adam is blond; or Eve is dark eyed and Adam either is not blond or is in love with Eve.

2-4. Your choice of sentence letters may of course vary from mine. But you should have sentence letters symbolizing the same atomic sentences as in the answers below.
a) RvT (R: Roses are red. T: Teller will eat his hat.)
b) P&-R (P: Monty Python is funny. R: Robert Redford is funny.)
c) -C&-N (C: Chicago is bigger than New York. N: New York is the largest city.)
d) FvD (F: I will finish this logic course. D: I will die trying to finish this logic course.)
e) -(H&S), or equally, -Hv-S (H: W.C. Fields is handsome. S: W.C. Fields is smart.)
f) -(GvU), or equally, -G&-U (G: Uncle Scrooge was generous. U: Uncle Scrooge was understanding.)
g) T&O (T: Minnesota Fats tried to diet. O: Minnesota Fats was very overweight.)
h) (P&I)&-E (P: Peter likes pickles. I: Peter likes ice cream. E: Peter likes to eat pickles and ice cream together.)
i) (R&B)&T (R: Roses are red. B: Violets are blue. T: Transcibing this jingle is not hard to do.)
j) (OvT)&(-S&-N), or equally, (OvT)&-(SvN) (O: Columbus sailed the ocean blue in 1491. T: Columbus sailed the ocean blue in 1492. S: Columbus discoverd the South Pole. N: Columbus discovered the North Pole.)
k) [(C&P)v(G&T)]&D (C:Luke will catch up with Darth Vader. P: Luke will put and end to Darth Vader. G: Darth Vader will get away. T: Darth Vader will cause more trouble. D: Eventually the Empire will be destroyed.)

ANSWERS TO EXERCISES IN VOLUME I, CHAPTER 3

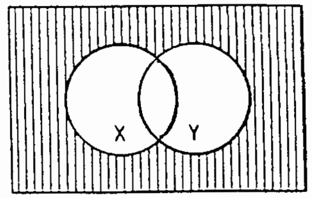
3-1. We are going to prove the DeMorgan law which says that -(XvY) is logically equivalent to -X&-Y. We will do this by calculating the Venn Diagram Area for the first sentence and then for the second sentence:

XvY



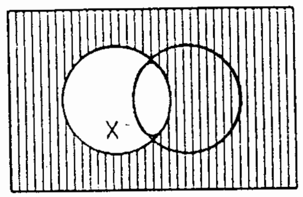
All points inside of X or inside of Y

-(XvY)



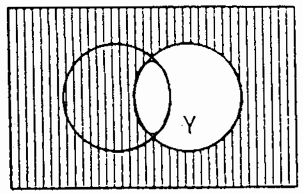
All points outside of (XvY)

-X



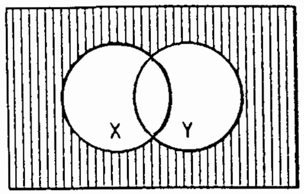
All points outside of X

-Y



All points outside of Y

-X&-Y

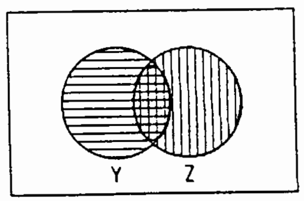


All points inside both -X and -Y

We can see that the areas are the same. Since the areas represent the cases in which the sentences are true, the two sentences are true in the same cases, that is, they are logically equivalent.

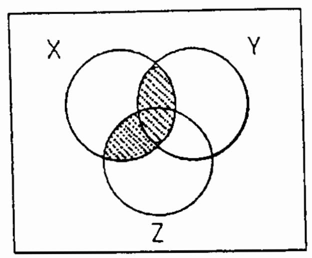
Now we use the same method to prove the logical equivalence of X&(YvZ) and (X&Y)v(X&Z):

YvZ



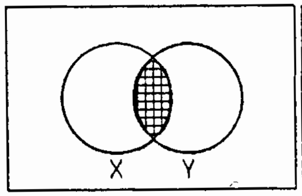
All points inside of Y or inside of Z

X&(YvZ)



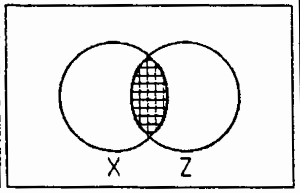
All points inside of both X and inside of YvZ.

X&Y



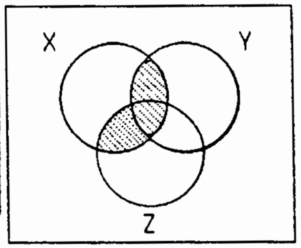
All points inside of both X and Y

X&Z



All points inside of both X and Z

(X&Y)v(X&Z)

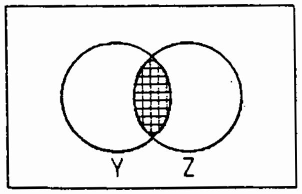


All points inside of either X&Y or X&Z.

Since the areas coincide, the sentences are logically equivalent.

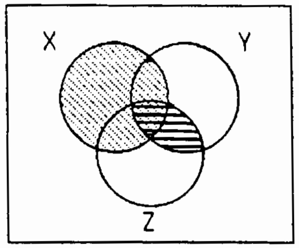
Finally we do the same for Xv(Y&Z) and (XvY)&(XvZ):

Y&Z



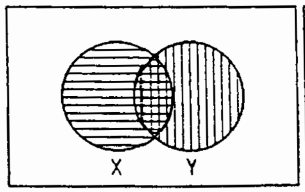
All points inside of both Y and Z

Xv(Y&Z)



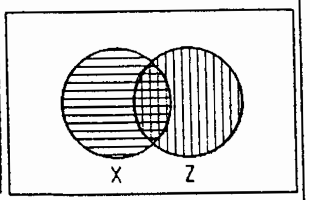
All points inside of X or inside of (Y&Z)

XvY



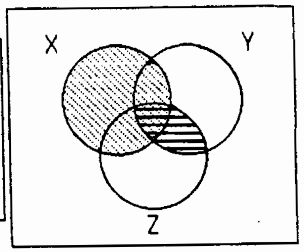
All points inside of X or inside of Y

XvZ



All points inside of X or inside of Z

(XvY)&(XvZ)



All points inside of of both XvY and of XvZ

3-2. a)

Bv-A	
-AvB	CM
-Av--B	DN, SLE
-(A&-B)	DM

b)

(A&B)vC	
Cv(A&B)	CM
(CvA)&(CvB)	D
(AvC)&(BvC)	C, SLE

c)

A&(--CvB)	
A&(CvB)	DN, SLE
(A&C)v(A&B)	D

d)

-[(A&-B)v(C&-B)]	
-(A&-B)&-(C&-B)	DM
(-Av--B)&(-Cv--B)	DM, SLE
(-AvB)&(-CvB)	DN, SLE
(-A&-C)vB	D, R

e)

(AvB)&(CvD)	
[(AvB)&C]v[(AvB)&D]	D
[(A&C)v(B&C)]v[(A&D)v(B&D)]	D, SLE
(A&C)v(B&C)v(A&D)v(B&D)	A

f)

(A&B)v(C&D)	
[(A&B)vC]&[(A&B)vD]	D
[(AvC)&(BvC)]&[(AvD)&(BvD)]	D, SLE
(AvC)&(BvC)&(AvD)&(BvD)	A

g)

(C&A)v(B&C)v[C&-(-B&-A)]	
(C&A)v(C&B)v[C&-(-B&-A)]	CM, SLE
[(C&A)v(C&B)]v[C&-(-B&-A)]	A
[C&(AvB)]v[C&-(-B&-A)]	D, SLE
[C&(AvB)]v[C&(--Bv--A)]	DM, SLE
[C&(AvB)]v[C&(BvA)]	DN, SLE
[C&(AvB)]v[C&(AvB)]	CM, SLE
C&(AvB)	R

h) Note that in proving two sentences, X and Y to be logically equivalent, one can just as well start with the second and prove it logically equivalent to the first as start with the first and prove it logically equivalent to the second.

C&[-Av-(-CvA)]	
C&[-Av(--C&-A)]	DM. SLE
C&[-Av(C&-A)]	DN, SLE
(C&-A)v[C&(C&-A)]	D
(C&-A)v[(C&C)&-A]	A, SLE
(C&-A)v(C&-A)	R, SLE
C&-A	R

i)

C&{-(Av-B)v[B&-(-CvA)]}	
C&{(-A&--B)v[B&(--C&-A)]}	DM, SLE
C&{(-A&B)v[B&(C&-A)]}	DN, SLE
C&{(-A&B)v{(B&C)&-A]}	A, SLE
[C&(-A&B)]v{C&[(B&C)&-A]}	D
[C&(-A&B)]v{[C&(B&C)]&-A}	A, SLE
[C&(-A&B)]v{[(B&C)&C]&-A}	CM, SLE
[C&(-A&B)]v{[B&(C&C)]&-A}	A, SLE
[C&(-A&B)]v{[B&C]&-A}	R, SLE
[(-A&B)&C]v{-A&[B&C]}	CM, SLE
(-A&B&C)v(-A&B&C)	A, SLE
-A&B&C	R

3-3. For any sentences X, Y, and Z, -(X&Y&Z) is logically equivalent to -Xv-Yv-Z. And -(XvYvZ) is logically equivalent to -X&-Y&-Z.

3-4. Suppose we have a disjunction XvY, and suppose that X is a logical truth. Than means that in every possible case X is true. But the disjunction XvY is true in any case in which either one of its disjunctions is true. Since X is always true, XvY is always true, which is to say that XvY is a logical truth.

Suppose, now, that we have a conjunction, X&Y, and that X is a contradiction. That is to say, in every possible case, X is false. But the conjunction X&Y is false in any case in which either of its conjuncts is false. Since X is false in every possible case, X&Y is false in every possible case, which means that X&Y is a contradiction.

3-5. By LTC 'A&(Bv-B)' is logically equivalent to 'A'.

3-6. a)

A&(-AvB)	
(A&-A)v(A&B)	D
A&B	CD (Law of contradictory disjunct)

b)

Av(-A&B)	
(Av-A)&(AvB)	D
AvB	LTC (Law of logically true conjunct)

c)

(A&B)v(A&-B)	
A&(Bv-B)	D
A	LTC

d)

(AvB)&(Av-B)	
Av(B&-B)	D
A	CD

e)

A&[Bv(-A&C)]		
(A&B)v[A&(-A&C)]	D	
(A&B)v[(A&-A)&C]	A, SLE	
(A&B)	CD	('(A&-A)&C' is a contradiction by the results of problem 3-4)

f)

CvB	
(C&A)v(C&-A)v(B&A)v(B&-A)	Law of expansion (problem c above) applied to 'C' and to 'B', SLE
(C&A)v(B&A)v(C&-A)v(B&-A)	CM, SLE

g) You can do this problem using problem d in the same way that we d f using c. Or,

(CvA)&(Bv-D)&(-AvC)&(DvB)	
(CvA)&(Cv-A)&(BvD)&(Bv-D)	CM, SLE
[Cv(A&-A)]&[Bv(D&-D)]	D
C&B	CD

h)

(A&B)v(-A&-B)	
(Av-A)&(Bv-A)&(Av-B)&(Bv-B)	D (Problem 3-2 f)
(Bv-A)&(Av-B)	LTC
(-AvB)&(-BvA)	CM, SLE

i)

-(-AvB)v-AvC	
(--A&-B)v-AvC	DM, SLE
(A&-B)v-AvC	DN, SLE
[(A&-B)v-A]vC	A, SLE
[(Av-A)&(-Bv-A)]vC	D, SLE
(-Bv-A)vC	LTC, SLE
(-Av-B)vC	CM, SLE
-Av-BvC	A

3-7. a) Neither. b) Contradiction. c) Neither. d) Logical Truth. e) Contradiction. f) Contradiction. g) Neither. h) Logical Truth. i) Logical Truth. j) Logical Truth. k) Neither. l) Logical Truth.

3-8. a) We first work the problem with a truth table:

A	B	A&B	-(A&B)
t	t	t	f
t	f	f	t
f	t	f	t
f	f	f	t

'-(A&B)' is true if case 2 is true, if case 3 is true or if case 4 is true. 'A&-B' says that case 2 is true, '-A&B' says that case 3 is true and '-A&-B' says that case 4 is true. So, the disjunction of these three case-describing sentences is logically equivalent to '-(A&B)':

The disjunctive normal form of '-(A&B)'is '(A&-B)v(-A&B)v(-A&-B)'.

Now let's try to work the problem by using a sequence of laws of logical equivalence. To work the problem given what you know, you have to apply DeMorgan's law and then fiddle around until you get the sentence into the right form. The following does the job:

-(A&B)	
-Av-B	DM
[(-A&B)v(-A&-B)]v[(-B&A)v(-B&-A)]	Law of expansion (problem 3-6 c), SLE
(-A&B)v(-A&-B)v(-B&A)	A, R

b) First with truth tables:

A	B	C	-A	A&B	-A&C	(A&B)v(-A&C)	-[(A&B)v(-A&C)]
t	t	t	f	t	f	t	f
t	t	f	f	t	f	t	f
t	f	t	f	f	f	f	t
t	f	f	f	f	f	f	t
f	t	t	t	f	t	t	f
f	t	f	t	f	f	f	t
f	f	t	t	f	t	t	f
f	f	f	t	f	f	f	t

There are four cases in which the final sentence is true, described by the sentences 'A&-B&C', 'A&-B&-C', '-A&B&-C', and '-A&-B&-C'. The whole sentence is true in any one of these cases, and in no others; so we get a logically equivalent sentence by taking the disjunction: The disjunctive normal form of' -[(A&B)v(-A&C)]' is '(A&-B&C)v(-A&B&-C)v(A&-B&-C)v(-A&-B&-C)'.

Now let's use laws of logical equivalence. I first apply DeMorgan's law twice. This gives a conjunction of disjunctions, and I want the opposite, namely a disjunction of conjunctions. But I can convert the one to the other by using the distributive law twice. Actually we already did the work in problem 3-4. e), so I just cite that problem for the result. The resulting conjunctions only involve two of the three letters in the problem, so I have to apply the law of expansion to each of them to get the final disjunctive normal form:

-[(A&B)v(-A&C)]	
-(A&B)&-(-A&C)	DM
(-Av-B)&(Av-C)	DM, DN, SLE
(-A&A)v(-B&A)v(-A&-C)v(-B&-C)	Problem 3-4. e)
(-B&A)v(-A&-C)v(-B&-C)	CD
(-B&A&C)v(-B&A&-C)v(-A&-C&B)v(-A&-C&-B)v(-B&-C&A)v(-B&-C&-A)	law of expansion
(-B&A&C)v(-B&A&-C)v(-A&-C&B)v(-A&-C&-B)	A, R.

3-9. If 'v' occurs in a sentence, it must occur with two disjuncts, in the form XvY. That is, XvY is either the whole sentence, or it is some subsentence. By the law of double negation XvY is logically equivalent to --(XvY), which by DeMorgan's law is equivalent to -(-X&-Y). So by the law of substitution of logical equivalents we can substitute -(-X&-Y) for XvY in the original sentence. We now repeat this procedure for all ocurrences of 'v' until all these occurences are eliminated.

To eliminate all occurences of '&' from a sentence we proceed in the same way except that we use the logical equivalences:

X&Y	
--(X&Y)	DN
-(-Xv-Y)	DM

3-10. For any sentences, X and Y, X*X is logically equivalent to -X, and X*Y is logically equivalen to -(X&Y). So (X*Y)•(X*Y) is logically eqivlalent to X&Y. Since we know that any truth function can be expressed with '&' and '-', and since we can express these with '*', we can express any truth function with '*', i.e., '*' is expressively complete.

3-11. The truth function given by the logical truth 'Av-A' cannot be expressed using '&' as the only connective. For suppose we had some big, long conjunction of conjunction of....conjunctions. At the outermost level this sentence will have the form X&Y, where X and Y may themselves be conjunctions. X&Y can be made false if we make X false. Now, Y is either an atomic sentence letter, in which case we can make it false, thereby making X&Y false. Or Y is a conjunction, X'&Y'. We can make the conjunction false just by making Y' false. Y' is either an atomic sentence letter which we can make false, or Y' is in turn a conjunction. We continue in this way, and sooner or later we get down to an atomic sentence letter. By making this bottom level sentence letter false, we make everything on up false. Thus there is a case for which X&Y is false, so that X&Y cannot be a logical truth.

For showing the expressive incompleteness of 'v' we proceed in the same way, to show that no disjunction of disjunction of...disjunctions can be a contradiction. At the bottom level, if we make one of the atomic sentence letters true, we make everything on up true.

'-' cannot express any truth function using two atomic sentence letters as arguments because '-' applies only to one component sentence.

4-1. a) All people die someday.
I am a person.
____
I will die someday.

b) All Republicans are conservative.
Reagan is a Republican.
____
Reagan is conservative.

(This argument is not sound because the first premise is not true.)

c) All Republicans are bald.
Reagan is a Republican.
____
Reagan is bald.

d) All ice cream is sweet.
This cookie is sweet.
____
This cookie is (made of) ice cream.

e) Anyone who loves logic is bald.
Robert Redford is bald.
____
Robert Redford loves logic.

4-2. a)

A	B	A&B	-(A&B)	-A	-B	
t	t	t	f	f	f	
t	f	f	t	f	t	
f	t	f	t	t	f *	CE
f	f	f	t	t	t *	

Since there is a counterexample the argument is INVALID.

b)

A	B	-A	-AvB
t	t	f	t *
t	f	f	f
f	t	t	t
f	f	t	t

Since there are no counterexamples the argument is VALID.

c)

A	B	-B	AvB	-BvA
t	t	f	t	t*
t	f	t	t	t*
f	t	f	t	f
f	f	t	f	t

Since there are no counterexamples, the argument is VALID.

d)

A	B	-A	AvB	-AvB	
t	t	f	t	t*	
t	f	f	t	f	
f	t	t	t	t*	CE
f	f	t	f	t	

Since there is a counterexample, the argument is INVALID.

e)

A	B	C	-C	A&B	A&-C	Bv-C	(A&B)v(A&-C)
t	t	t	f	t	f	t	t*
t	t	f	t	t	t	t	t*
t	f	t	f	f	f	f	f
t	f	f	t	f	t	t	t*
f	t	t	f	f	f	t	f
f	t	f	t	f	f	t	f
f	f	t	f	f	f	f	f
f	f	f	t	f	f	t	f

Since there are no counterexamples, the argument is VALID.

4-3. First, suppose that X is logically equivalent to Y. That means that in all possible cases X and Y have the same truth value. In particular, in all cases in which X is true Y is true also, which is to say that the argument X/Y is valid. Similarly, in all possible cases in which Y is true X is true also, which is to say that the argument Y/X is valid.

Now suppose that the argument X/Y and Y/X are both valid. We have to show that on this assumption X and Y have the same truth value in all possible cases. Well, suppose they don't. That is, suppose there is a possible case in which X is true and Y is false or a possible case in which Y is true and X is false. A possible case of the first kind would be a counterexample to the argument X/Y, and a possible case of the second kind would be a counterexample to the argument Y/X. However, we are supposing that these two arguments are valid. So there can't be any such possible cases and X and Y have the same truth value in all possible cases.

4-4. Formation Rules:

i) Every capital letter 'A', 'B', 'C' ... is a sentence of sentence logic. Such a sentence is called an Atomic Sentence or a Sentence Letter.

ii) If X is a sentence of sentence logic, so is (-X), that is, the sentence formed by taking X, writing a '-' in front of it, and surrounding the whole by parentheses. Such a sentence is called a Negated Sentence.

iii) If X, Y,...,Z are sentences of sentence logic, so is (X&Y&...&Z), that is the sentence formed by writing all of the sentences X,Y,...,Z separated by '&'s and surrounding the whole with parentheses. Such a sentence is called a Conjunction, and the sentences X, Y,...Z are called its Conjuncts.

iv) If X, Y,...,Z are sentences of sentence logic, so is (XvYv...vZ), that is the sentence formed by writing all of the sentences X,Y,...,Z separated by 'v's and surrounding the whole with parentheses. Such a sentence is called a Disjunction, and the sentences X, Y,...Z are called its Disjuncts.

v) If X and Y are sentences of sentence logic, so is (X -> Y), that is the sentence formed by writing X, followed by '->', followed by Y and surrounding the whole with parentheses. Such a sentence is called a Conditional. X is called the conditional's Antecedent and Y its Consequent.

vi) If X and Y are sentences of sentence logic, so is (X <-> Y), that is the sentence formed by writing X, followed by '<->', followed by Y and surrounding the whole with parentheses. Such a sentence is called a Biconditional. X and Y are called the biconditional's Components.

vii) Only those expressons formed using rules (i)-(vi) are sentences of sentence logic.

Rules of Valuation:

i) The truth value of a negated sentence is 't' if the component (the sentence which has been negated) is 'f'. The truth value of a negated sentence is 'f' if the truth value of the component is 't'.

ii) The truth value of a conjunction is 't' if all conjuncts have truth value 't'. Otherwise the truth value of the conjunction is 'f'.

iii) The truth value of a disjunction is 't' if at least one of the disjuncts have truth value 't'. Otherwise the truth value of the disjunction is 'f'.

iv) The truth value of a conditional is 'f' if its antecedent is true and its consequent is false. Otherwise the truth value of the conditional is 't'.

v) The truth value of a biconditional is 't' if both compenents have the same truth value. Otherwise the truth value of the biconditional is 'f'.

4-5.

a)

A	B	A->B	
t	t	t*	
t	f	f	
f	t	t*	CE
f	f	t	

Since there is a counterexample, the argument is INVALID.

b)

A	B	-B	A->-B	-A
t	t	f	f	f
t	f	t	t	f
f	t	f	t	t*
f	f	t	t	t

Since there are no counterexamples, the argument is VALID.

c)

A	B	-B	A<->B	Av-B	
t	t	f	t	t*	
t	f	t	f	t	
f	t	f	f	f	
f	f	t	t	t*	CE

Since there is a counterexample, the argument is INVALID.

d)

A	B	-B	A<->-B	Av-B	AvB
t	t	f	f	t	t
t	f	t	t	t	t*
f	t	f	t	f	t
f	f	t	f	t	f

Since there are no counterexamples, the argument is VALID.

e)

A	B	C	AvB	A&C	CvA	(AvB)->(A&C)	-C	
t	t	t	t	t	t	t	f *	CE
t	t	f	t	f	t	f	t	
t	f	t	t	t	t	t	f *	CE
t	f	f	t	f	t	f	t	
f	t	t	t	f	t	f	f	
f	t	f	t	f	f	f	t	
f	f	t	f	f	t	t	f *	CE
f	f	f	f	f	f	t	t	

Since there are counter-examples to the argument, the argument is INVALID.

f)

A	B	C	-B	-C	AvB	Av-C	(AvB)<->(Av-C)	-BvC	AvC	
t	t	t	f	f	t	t	t	t	t	*
t	t	f	f	t	t	t	t	f	t	
t	f	t	t	f	t	t	t	t	t	*
t	f	f	t	t	t	t	t	t	t	*
f	t	t	f	f	t	f	f	t	t	
f	t	f	f	t	t	t	t	f	f	
f	f	t	t	f	f	f	t	t	t	*
f	f	f	t	t	f	t	f	t	f	

Since there are no counterexamples to the argument, the argument is VALID.

4-6.

a) (A->B)<->(-B->-A)
(A->B)<->(A->B) CP, SLE

This is a logical truth, since A->B is logically equivalent to itself, and a biconditional is a logical truth if its components are logically equivalent.

b) (A<->-A)->(B<->B)
-(A<->-A)v(B<->B) C

But 'B<->B' is a logical truth (as in problem a), and a disjunction with a logical truth as a disjunct is a logical truth. So the original sentence is a logical truth.

c) A<->-A
(A->-A)&(-A->A) B
(-Av-A)&(--AvA) C
-A&A DN, R, SLE

-A&A is a contradiction. So A<->-A is a contradiction.

d) A->-B
-Av-B C (neither a logical truth nor a contradicition.)

e) (A->B)v(A->-B)
-AvBv-Av-B C, A, SLE
-Av(Bv-B) R, A, SLE

A disjunction with a logically true disjunct is logically true (exercise 3-4). So this is a logical truth.

f) -(AvB)&(-A->B)
-(AvB)&-(-A&-B) C, SLE
-(AvB)&(AvB) DM, DN, SLE

This is a conjunction of a sentence with it's negation and so is a contradiction.

g) (A<->A)->(B<->-B): Problem (c) shows that the consequent is a contradiction. Since 'A' is logically equivalent to itself, the antecedent is a logical truth. Since the antecedent is always true and the consequent is always false, this is a contradiction.

h) [-(B->A)&(C->A)]->(C->B)
-[-(B->A)&(C->A)]v(C->B) C
(B->A)v-(C->A)v(C->B) DM, SLE
(-BvA)v(C&-A)v(-CvB) C, SLE
(-BvA)v(-CvB)v(C&-A) A, SLE
(-BvB)v(Av-C)v(C&-A) CM, A, SLE

This is a logical truth because one of its disjuncts is a logical truth.

i) [A->(B->C)]->[(A->B)->(A->C)]
-(-Av-BvC)v[-(-AvB)v-AvC] CD, A, SLE
(A&B&-C)v(A&-B)v-AvC DM, DN, A, SLE
[Cv(A&B&-C)]v[-Av(A&-B)] A,
{[Cv(A&B)]&(Cv-C)}v [(-AvA)&(-Av-B)] D, SLE
[Cv(A&B)]v-Av-B LTC, A, SLE
Cv[(A&B)v-(AvB)] DM, A, SLE

This is a logical truth because one of its disjuncts is a logical truth.

4-7.

a) 'You will sleep too late unless you set your alarm.' This sentence makes a causal claim, to the effect that setting your alarm will keep you from sleeping too late. Whether or not the causal connection holds is a matter above and beyond the truth value of the compenents, 'You will sleep too late,' and 'You set your alarm.'

b) 'Argument 4-5.a) is valid unless it has a counterexample'. Since, by definition, an argument is valid if and only if it has no counterexamples, only the truth value of the compenents is relevant to the truth value of this example.

c) i) 'You won't get an A in this course unless you study hard.' This transcribes as 'Sv-A', (S: You study hard. A: You get an A in this course.)

ii) The critically ill patient will die unless the doctors operate. This transcribes as '-O->D' (O: The doctors operate. D: The critically ill patient will die.)

iii) 'You may drive through the intersection unless there is a red light.' D<->-R (D: You may drive through the intersection. R: There is a red light.)

These examples, and the whole business of transcribing 'unless' require some comment. First, most logic texts will tell you to transcirbe 'unless' as a conditional or a disjunction. By the law of the conditional, these two approaches are interchangeble, sometimes the one being more natural, sometimes the other. Few even mention the possibility of transcribing with the biconditional. I, in a minority, think that in most cases the biconditional supplies a more faithful transcription.

Notice that there is a strong parallel between the difference in transcribing 'unless' with a conditional versus a biconditional and the difference in transcribing 'or' as inclusive versus exculsive disjunction. The parellel becomes more striking when you note that 'X<->-Y' expresses the exclusive disjunction of X and Y. Thus transcribing X unless Y as -Y->X, equivalently as XvY, corresponds to transcribing 'unless' as inclusive 'or'; while transcribing it as X<->-Y corresponds to transcribing 'unless' as exclusive 'or'.

The whole issue is really very unclear, and the reason is that in almost all usages, 'unless' is not truth functional. Thus, like a great many uses of 'If...then...', transciptions into logic are faulty and inaccurate at best.

4-8.
a) D->-A
b) D->A
c) A->-E
d) E->-B
e) A<->D
f) B->E (Possibly 'B<->E' is a correct transcription)
g) C->A (Possibly 'C<->A' is a correct transcription)
h) -AvB (Possibly 'A<->B'. 'Unless' is tough!)
i) Cv-E (Possibly 'C<->E')
j) B->(A->D)
k) -B->A (Or, equivalently, '-B->[(D->A)&(-D->A)]')
l) (B&A)<->C
m) (C&E)->B

4-9. The following words can be used in various ways to build up non-truth functional connectives in English. For example, 'believes', when combined with a person's name makes such a connective: 'John believes that...'. 'Ought' is the crucial word in the connective 'It ought to be the case that...' and so on.

i) 'makes', 'causes', 'because', 'brings about' 'induces', 'since', 'means', 'indicates'.

ii) 'must', 'possible', 'probable', 'more probable (likely) than', 'ought', 'may'.

iii) 'believe', 'hope', 'want', 'wish', 'fear', 'expect', 'like', 'encourage', 'discourage'.

iv) 'before', 'sooner than', 'after', 'later than', 'at the same time as', 'since'.

There are, of course, many, many other such connectives in English!

5-1. a)

1	-P>-D	P
2	-D>-F	P
3	-P	P
4	-D	1,3,>E
5	-F	2,4,>E

5-1. b)

1	-C>-D	P
2	-C	P
3	-D	1,2,>E
4	-DvE	3,vI

5-1. c)

1	Fv-G	P
2	-F	P
3	GvK	P
4	-G	1,2,vE
5	K	3,4,vE

5-1. d)

1	A>B	P
2	A	P
3	B>-C	P
4	CvD	P
5	B	1,2,>E
6	-C	3,5,>E
7	D	4,6,vE
8	DvE	7,vI

5-1. e)

1	Lv-M	P
2	-L	P
3	MvD	P
4	D>H	P
5	-M	1,2,vE
6	D	3,5,vE
7	H	4,6,>E

5-1. f)

1	C	P
2	C>(HvA)	P
3	-H	P
4	HvA	1,2,>E
5	A	3,4,vE
6	Av-K	5,vI

5-1. g)

1	(Kv-D)>F	P
2	K	P
3	Kv-D	2,vI
4	F	1,3,>E
5	FvD	4,vI

5-1. h)

1	D	P
2	(DvB)>-G	P
3	(-Gv-H)>(GvQ)	P
4	DvB	1,vI
5	-G	2,4,>E
6	-Gv-H	5,vI
7	GvQ	3,6,>E
8	Q	5,7,vE
9	Qv-A	8,vI

5-1. i)

1	(Mv-T)>(AvJ)	P
2	-A	P
3	BvM	P
4	-A>-B	P
5	-B	2,4,>E
6	M	3,5,vE
7	Mv-T	6,vI
8	AvJ	1,7,>E
9	J	2,8,vE
10	JvD	9,vI

5-2. a)

```
1   | A>B              P
2   | B>C              P
3   | C>D              P
    |-----------------
4   | | A              A
    | |---------------
5   | | A>B            1,R
6   | | B              4,5,>E
7   | | B>C            2,R
8   | | C              6,7,>E
9   | | C>D            3,R
10  | | D              8,9,>E
11  | A>D              4-10,>I
```

5-2. b)

```
1   | NvP              P
    |-----------------
2   | | -N             A
    | |---------------
3   | | NvP            1,R
4   | | P              2,3,vE
5   | -N>P             2-4,>I
```

5-2. c)

```
1   | B                P
    |-----------------
2   | | A              A
    | |---------------
3   | | B              1,R
4   | A>B              2-3,>I
```

5-2. d)

```
1   | -B               P
    |-----------------
2   | | BvC            A
    | |---------------
3   | | -B             1,R
4   | | C              2,3,vE
5   | (BvC)>C          2-4,>I
```

5-2. e)

```
1   | K>-D             P
2   | DvH              P
    |-----------------
3   | | K              A
    | |---------------
4   | | K>-D           1,R
5   | | -D             3,4,>E
6   | | DvH            2,R
7   | | H              5,6,vE
8   | K>H              3-7,>I
```

5-2. f)

```
1   | A>B              P
    |-----------------
2   | | A              A
    | |---------------
3   | | A>B            1,R
4   | | B              2,3,>E
5   | | BvC            4,vI
6   | A>(BvC)          2-5,>I
```

5-2. g)

```
1   | F>(CvM)          P
2   | -C               P
    |-----------------
3   | | F              A
    | |---------------
4   | | F>(CvM)        1,R
5   | | CvM            3,4,>E
6   | | -C             2,R
7   | | M              5,6,vE
8   | F>M              3-7,>I
```

5-2. h)

```
1   | (DvB)>J          P
    |-----------------
2   | | D              A
    | |---------------
3   | | DvB            2,vI
4   | | (DvB)>J        1,R
5   | | J              3,4,>E
6   | D>J              2-5,>I
```


5-2. i)

```
1   | A>K              P
2   | (KvP)>L          P
    |-----------------
3   | | A              A
    | |---------------
4   | | A>K            1,R
5   | | K              3,4,>E
6   | | KvP            5,vI
7   | | (KvP)>L        2,R
8   | | L              6,7,>E
9   | A>L              3-8,>I
```

5-2. j)

```
1   | Q>-S             P
2   | Q>(SvF)          P
    |-----------------
3   | | Q              A
    | |---------------
4   | | Q>-S           1,R
5   | | -S             3,4,>E
6   | | Q>(SvF)        2,R
7   | | SvF            3,6,>E
8   | | F              5,7,vE
9   | Q>F              3-8,>I
```

5-2. k)

```
1   | P                P
2   | (-DvK)>B         P
3   | (Fv-D)>-K        P
4   | P>(Kv-F)         P
    |-----------------
5   | | Fv-D           A
    | |---------------
6   | | (Fv-D)>-K      3,R
7   | | -K             5,6,>E
8   | | P              1,R
9   | | P>(Kv-F)       4,R
10  | | Kv-F           8,9,>E
11  | | -F             7,10,vE
12  | | -D             5,11,vE
13  | | -DvK           12,vI
14  | | (-DvK)>B       2,R
15  | | B              13,14,>E
16  | | Bv-P           15,vI
17  | (Fv-D)>(Bv-P)    5-16,>I
```

5-3. a)

```
1   | B&(B>-A)         P
    |-----------------
2   | B                1,&E
3   | B>-A             1,&E
4   | -A               2,3,>E
```

5-3. b)

```
1   | -C<->(AvB)       P
2   | A                P
    |-----------------
3   | (AvB)>-C         1,<>E
4   | AvB              2,vI
5   | -C               3,4,>E
```

5-3. c)

```
1   | A>-B             P
2   | BvC              P
    |-----------------
3   | | A              A
    | |---------------
4   | | A>-B           1,R
5   | | -B             3,4,>E
6   | | BvC            2,R
7   | | C              5,6,vE
8   | A>C              3-7,>I
```

5-3. d)

```
1   | D                P
2   | (D&A)>C          P
    |-----------------
3   | | A              A
    | |---------------
4   | | D              1,R
5   | | D&A            3,4,&I
6   | | (D&A)>C        2,R
7   | | C              5,6,>E
8   | A>C              3-7,>I
```

5-3. e)

```
1   | AvB              P
    |-----------------
2   | | -A&-B          A
    | |---------------
3   | | -A             2,&E
4   | | -B             2,&E
5   | | AvB            1,R
6   | | B              3,5,vE
7   | -(-A&-B)         2-6,-I
```

5-3. f)

```
1   |A&B            P
    |---
2   | |A            A
    | |---
3   | |A&B          1,R
4   | |B            3,&E
5   |A>B            2-4,>I
6   | |B            A
    | |---
7   | |A&B          1,R
8   | |A            7,&E
9   |B>A            6-8,>I
10  |A<->B          5,9,<>I
```

5-3. g)

```
1   |-A>B           P
2   |-A>-B          P
    |---
3   | |-A           A
    | |---
4   | |-A>B         1,R
5   | |B            3,4,>E
6   | |-A>-B        2,R
7   | |-B           3,6,>E
8   |--A            3-7,-I
9   |A              8,-E
```

5-4. a)

```
1   |C&-H           P
    |---
2   |-H             1,&E
```

5-4. b)

```
1   |JvD            P
2   |---D           P
    |---
3   |-D             2,-E
4   |J              1,3,vE
```

5-4. c)

```
1   |A&B            P
    |---
2   |A              1,&E
3   |B              1,&E
4   |B&A            2,3,&I
```

5-4. d)

```
1   |A>-D           P
2   |--A            P
    |---
3   |A              2,-E
4   |-D             1,3,>E
```

5-4. e)

```
1   |G>D            P
2   |G>-D           P
    |---
3   | |G            A
    | |---
4   | |G>D          1,R
5   | |D            3,4,>E
6   | |G>-D         2,R
7   | |-D           3,6,>E
8   |-G             3-7,-I
```

5-4. f)

```
1   |A<->-B         P
    |---
2   |-B>A           1,<>E
```

5-4. g)

```
1   |M              P
2   |Rv-H           P
    |---
3   |M&(Rv-H)       1,2,&I
```

5-4. h)

```
1   |A&(B&C)        P
    |---
2   |A              1,&E
3   |B&C            1,&E
4   |B              3,&E
5   |C              3,&E
6   |A&B            2,4,&I
7   |(A&B)&C        5,6,&I
```

5-4. i)

```
1   |-C>D           P
2   |D>-C           P
    |---
3   |D<->-C         1,2,<>I
```

5-4. j)

```
1   |A<->-B         P
2   |-B             P
    |---
3   |-B>A           1,<>E
4   |A              2,3,>E
```

5-4. k)

```
1   |-C>---A        P
2   |-C             P
    |---
3   |---A           1,2,>E
4   |-A             3,-E
```

5-4. l)

```
1   |K>-B           P
2   |B&F            P
    |---
3   | |K            A
    | |---
4   | |K>-B         1,R
5   | |-B           3,4,>E
6   | |B&F          2,R
7   | |B            6,&E
8   |-K             3-7,-I
```

5-4. m)

```
1   |-P             P
2   |-Q             P
    |---
3   | |PvQ          A
    | |---
4   | |-P           1,R
5   | |Q            3,4,vE
6   | |-Q           2,R
7   |-(PvQ)         3-6,-I
```

5-4. n)

```
1   |(N>K)&(N>L)    P
    |---
2   | |N            A
    | |---
3   | |(N>K)&(N>L)  1,R
4   | |N>K          3,&E
5   | |N>L          3,&E
6   | |K            2,4,>E
7   | |L            2,5,>E
8   | |K&L          6,7,&I
9   |N>(K&L)        2-8,>I
```

5-4. o)

```
1   |D>(AvF)        P
2   |D>-F           P
3   |-A             P
    |---
4   | |D            A
    | |---
5   | |D>(AvF)      1,R
6   | |AvF          4,5,>E
7   | |-A           3,R
8   | |F            6,7,vE
9   | |D>-F         2,R
10  | |-F           4,9,>E
11  |-D             4-10,-I
12  |-D&-A          3,11,&I
```

5-4. p)

```
1   |H<->J          P
2   |H<->K          P
    |---
3   |H>J            1,<>E
4   |J>H            1,<>E
5   |H>K            2,<>E
6   |K>H            2,<>E
7   | |J            A
    | |---
8   | |J>H          4,R
9   | |H            7,8,>E
10  | |H>K          5,R
11  | |K            9,10,>E
12  |J>K            7-11,>I
13  | |K            A
    | |---
14  | |K>H          6,R
15  | |H            13,14,>E
16  | |H>J          3,R
17  | |J            15,16,>E
18  |K>J            13-17,>I
19  |J<->K          12,18,<>I
```

5-5.

```
| X&Y&Z
| X        &E

| X&Y&Z
| Y        &E

| X&Y&Z
| Z        &E

| X
| Y
| Z
| X&Y&Z    &I

| X
| XvYvZ    vI

| Y
| XvYvZ    vI

| Z
| XvYvZ    vI

| XvYvZ
| -X
| YvZ      vE

| XvYvZ
| -Y
| XvZ      vE

| XvYvZ
| -Z
| XvY      vE
```

5-6. For most valid arguments, some truth assignments to the sentence letters which make one or more premise false will make the conclusion true and some such assignments will make the conclusion false. For example, the argument, "A, A->B. Therefore B." is valid. Making A true and B false makes one premise, 'A->B',and the conclusions, 'B' false. Making 'A' false and 'B' true makes one premise, 'A' false and the conclusion 'B' true. Some valid arguments do not have so much freedom. "B. Therefore B." is valid. Obviously in this example the premise and conclusion always have the same truth value. "B. Therefore Av-A." is also valid. In this case the conclusion is a logical truth and so always true. But these are special cases. In general, anything can happen, depending on the details of the case.

6-1. a)

```
1   | Kv-I            P
    |----------------
2   |  | -K&I         A
    |  |-------------
3   |  | Kv-I         1,R
4   |  | -K           2,&E
5   |  | -I           3,4,vE
6   |  | I            2,&E
7   | -(-K&I)         2-6,-I
```

6-1. b)

```
1   | -C->A           P
2   | B->-A           P
3   | B               P
    |----------------
4   | -A              2,3,>E
5   |  | -C           A
    |  |-------------
6   |  | -C->A        1,R
7   |  | A            5,6,>E
8   |  | -A           4,R
9   | --C             5-8,-I
10  | C               9,-E
```

6-1. c)

```
1   | -D->-K          P
2   | K               P
3   | -KvH            P
    |----------------
4   | H               2,3,vE
5   |  | -D           A
    |  |-------------
6   |  | -D->-K       1,R
7   |  | -K           5,6,>E
8   |  | K            2,R
9   | --D             5-8,-I
10  | D               9,-E
11  | H & D           4,10,&I
```

6-1. d)

```
1   | -F->G           P
2   | G->-E           P
    |----------------
3   |  | E            A
    |  |-------------
4   |  |  | G         A
    |  |  |----------
5   |  |  | G->-E     2,R
6   |  |  | -E        4,5,>E
7   |  |  | E         3,R
8   |  | -G           4-7,-I
9   |  |  | -F        A
    |  |  |----------
10  |  |  | -F->G     1,R
11  |  |  | G         9,10,>E
12  |  |  | -G        8,R
13  |  | --F          9-12,-I
14  |  | F            13,-E
15  | E->F            3-14,>I
```

6-1. e)

```
1   | A<->-B          P
    |----------------
2   |  | -A           A
    |  |-------------
3   |  | A<->-B       1,R
4   |  | -B->A        3,<->E
5   |  |  | -B        A
    |  |  |----------
6   |  |  | -B->A     4,R
7   |  |  | A         5,6,>E
8   |  |  | -A        2,R
9   |  | --B          5-8,-I
10  |  | B            9,-E
11  | -A->B           2-10,>I
```

6-1. f)

```
1   | AvB                P
2   | B>C                P
3   | -CvD               P
4   | | -D               A
5   | | -CvD             3,R
6   | | -C               4,5,vE
7   | | | B              A
8   | | | B>C            2,R
9   | | | C              7,8,>E
10  | | | -C             6,R
11  | | -B               7-10,-I
12  | | AvB              1,R
13  | | A                11,12,vE
14  | -D>A               4-13,>I
```

6-2. a)

```
1   | (AvB)&((AvB)>C)    P
2   | AvB                1,&E
3   | (AvB)>C            1,&E
4   | C                  2,3,>E
```

6-2. b)

```
1   | A                          P
2   | (AvB)<->((A>K)&(B>K))      P
3   | (AvB)>((A>K)&(B>K))        2,<>E
4   | AvB                        1,vI
5   | (A>K)&(B>K)                3,4,>E
6   | A>K                        5,&E
7   | K                          1,6,>E
```

6-2. c)

```
1   | [A>(Dv-B)]&{[A>(Dv-B)]>(B>A)}   P
2   | A>(Dv-B)                        1,&E
3   | (A>(Dv-B))>(B>A)                1,&E
4   | B>A                             2,3,>E
```

6-3. a)

```
1   | R                  P
2   | RvD                1,vI
3   | RvK                1,vI
4   | (RvD)&(RvK)        2,3,&I
```

6-3. b)

```
1   | (-A&B)vC           P
2   | AvD                P
3   | | -C               A
4   | | (-A&B)vC         1,R
5   | | -A&B             3,4,vE
6   | | -A               5,&E
7   | | AvD              2,R
8   | | D                6,7,vE
9   | -C>D               3-8,>I
```

6-3. c)

```
1   | -(Hv-D)            P
2   | F>H                P
3   | | F                A
4   | | F>H              2,R
5   | | H                3,4,>E
6   | | Hv-D             5,vI
7   | | -(Hv-D)          1,R
8   | -F                 3-7,-I
```

6-3. d)

```
1   | F>(O>M)            P
2   | | F&O              A
3   | | F                2,&E
4   | | F>(O>M)          1,R
5   | | O>M              3,4,>E
6   | | O                2,&E
7   | | M                5,6,>E
8   | (F&O)>M            2-7,>I
```


6-3. e)

```
1   | P&-Q               P
2   | | -R               A
3   | | | P>(RvQ)        A
4   | | | P&-Q           1,R
5   | | | P              4,&E
6   | | | RvQ            3,5,>E
7   | | | -R             2,R
8   | | | Q              6,7,vE
9   | | | -Q             4,&E
10  | | -(P>(RvQ))       3-9,-I
11  | -R>-(P>(RvQ))      2-10,>I
```

6-3. f)

```
1   | (K&G)>S            P
2   | | K                A
3   | | | G              A
4   | | | (K&G)>S        1,R
5   | | | K              2,R
6   | | | K&G            3,5,&I
7   | | | S              4,6,>E
8   | | G>S              3-7,>I
9   | K>(G>S)            2-8,>I
```

6-3. g)

```
1   | -(A&-F)            P
2   | D>A                P
3   | | D                A
4   | | | -F             A
5   | | | D>A            2,R
6   | | | D              3,R
7   | | | A              5,6,>E
8   | | | A&-F           4,7,&I
9   | | | -(A&-F)        1,R
10  | | --F              4-9,-I
11  | | F                10,-E
12  | D>F                3-11,>I
```

6-3. h)

```
1   | -(N>I)             P
2   | -I>C               P
3   | | I                A
4   | | | N              A
5   | | | I              3,R
6   | | N>I              4-5,>I
7   | | -(N>I)           1,R
8   | -I                 3-7,-I
9   | C                  2,8,>E
```

6-3. i)

```
1   | -M>-L              P
2   | -L>-K              P
3   | | K                A
4   | | | -M             A
5   | | | -M>-L          1,R
6   | | | -L             4,5,>E
7   | | | -L>-K          2,R
8   | | | -K             6,7,>E
9   | | | K              3,R
10  | | --M              4-9,-I
11  | | M                10,-E
12  | K>M                3-11,>I
```

6-3. j)

```
1   | QvF                P
2   | Q>A                P
3   | F>A                P
4   | | -A               A
5   | | | Q              A
6   | | | Q>A            2,R
7   | | | A              5,6,>E
8   | | | -A             4,R
9   | | -Q               5-8,-I
10  | | QvF              1,R
11  | | F                9,10,vE
12  | | F>A              3,R
13  | | A                11,12,>E
14  | --A                4-13,-I
15  | A                  14,-E
```

6-3. k)

```
1   |-(S&T)                P
2   |SvT                   P
    |----------------------
3   | |-S                  A
    | |--------------------
4   | |SvT                 2,R
5   | |T                   3,4,vE
6   |-S>T                  3-5,>I
7   | |T                   A
    | |--------------------
8   | | |S                 A
    | | |------------------
9   | | |T                 7,R
10  | | |S&T               8,9,&I
11  | | |-(S&T)            1,R
12  | |-S                  8-11,-I
13  |T>-S                  7-12,>I
14  |-S<->T                6,13,<>I
```

6-3. l)

```
1   |-C>(AvB)              P
2   |-D>(Cv-B)             P
3   |-(CvD)                P
    |----------------------
4   | |C                   A
    | |--------------------
5   | |CvD                 4,vI
6   | |-(CvD)              3,R
7   |-C                    4-6,-I
8   |AvB                   1,7,>E
9   | |D                   A
    | |--------------------
10  | |CvD                 9,vI
11  | |-(CvD)              3,R
12  |-D                    9-11,-I
13  |Cv-B                  2,12,>E
14  |(AvB)&(Cv-B)          8,13,&I
```

6-3. m)

```
1   |G<->-H                P
    |----------------------
2   |G>-H                  1,<>E
3   |-H>G                  1,<>E
4   | |-G                  A
    | |--------------------
5   | | |-H                A
    | | |------------------
6   | | |-H>G              3,R
7   | | |G                 5,6,>E
8   | | |-G                4,R
9   | |--H                 5-8,-I
10  | |H                   9,--E
11  |-G>H                  4-10,>I
12  | |H                   A
    | |--------------------
13  | | |G                 A
    | | |------------------
14  | | |G>-H              2,R
15  | | |-H                13,14,>E
16  | | |H                 12,R
17  | |-G                  13-16,-I
18  |H>-G                  12-17,>I
19  |-G<->H                11,18,<>I
```

6-3. n)

```
1   |P<->Q                 P
    |----------------------
2   | |P<->-Q              A
    | |--------------------
3   | | |P                 A
    | | |------------------
4   | | |P<->Q             1,R
5   | | |P>Q               4,<>E
6   | | |Q                 3,5,>E
7   | | |P<->-Q            2,R
8   | | |P>-Q              7,<>E
9   | | |-Q                3,8,>E
10  | |-P                  3-9,-I
11  | |-Q>P                2,<>E
12  | | |-Q                A
    | | |------------------
13  | | |-Q>P              11,R
14  | | |P                 12,13,>E
15  | | |-P                10,R
16  | |--Q                 12-15,-I
17  | |Q                   16,--E
18  | |P<->Q               1,R
19  | |Q>P                 18,<>E
20  | |P                   17,19,>E
21  |-(P<->-Q)             2-20,-I
```


6-3. o)

```
1   |(N>S)&(G>D)                        P
2   |(SvD)>{[F>(FvK)]>(N&G)}            P
    |-----------------------------------
3   |N>S                                1,&E
4   |G>D                                1,&E
5   | |N                                A
    | |---------------
6   | |N>S                              3,R
7   | |S                                5,6,>E
8   | |SvD                              7,vI
9   | |(SvD)>((F>(FvK))>(N&G))          2,R
10  | |(F>(FvK))>(N&G)                  8,9,>E
11  | | |F                              A
    | | |-------------
12  | | |FvK                            11,vI
13  | |F>(FvK)                          11-12,>I
14  | |N&G                              10,13,>E
15  | |G                                14,&E
16  |N>G                                5-15,>I
17  | |G                                A
    | |---------------
18  | |G>D                              4,R
19  | |D                                17,18,>E
20  | |SvD                              19,vI
21  | |(SvD)>((F>(FvK))>(N&G))          2,R
22  | |(F>(FvK))>(N&G)                  20,21,>E
23  | | |F                              A
    | | |-------------
24  | | |FvK                            23,vI
25  | |F>(FvK)                          23-24,>I
26  | |N&G                              22,25,>E
27  | |N                                26,&E
28  |G>N                                17-27,>I
29  |N<->G                              16,28,<>I
```

6-3. p)

```
1   |(S&B)>K               P
2   |(G>P)&(GvP)           P
3   |-B<->(-P&G)           P
    |----------------------
4   | |S                   A
    | |--------------------
5   | | |-B                A
    | | |------------------
6   | | |-B<->(-P&G)       3,R
7   | | |-B>(-P&G)        6,<>E
8   | | |-P&G              5,7,>E
9   | | |-P                8,&E
10  | | |G                 8,&E
11  | | |(G>P)&(GvP)       2,R
12  | | |G>P               11,&E
13  | | |P                 10,12,>E
14  | |--B                 5-13,-I
15  | |B                   14,--E
16  | |S&B                 4,15,&I
17  | |(S&B)>K             1,R
18  | |K                   16,17,>E
19  |S>K                   4-18,>I
```

6-3. q)

```
1   |CvB                 P
2   |-(C&-B)             P
3   |-(-C&B)             P
4   | |-C                A
5   | |CvB               1,R
6   | |B                 4,5,vE
7   | |-C&B              4,6,&I
8   | |-(-C&B)           3,R
9   |--C                 4-8,-I
10  |C                   9,-E
11  | |-(Bv-C)           A
12  | | |B               A
13  | | |Bv-C            12,vI
14  | | |-(Bv-C)         11,R
15  | |-B                12-14,-I
16  | | |-C              A
17  | | |Bv-C            16,vI
18  | | |-(Bv-C)         11,R
19  | |--C               16-18,-I
20  | |C                 19,-E
21  | |C&-B              15,20,&I
22  | |-(C&-B)           2,R
23  |--(Bv-C)            11-22,-I
24  |Bv-C                23,-E
25  |C&(Bv-C)            10,24,&I
```

6-3. r)

```
1   |H>(D>K)             P
2   |(K&M)>P             P
3   |D>-(M>P)            P
4   | |H                 A
5   | | |D               A
6   | | | |I             A
7   | | | |D>-(M>P)     3,R
8   | | | |-(M>P)       6,7,>E
9   | | | |H            4,R
10  | | | |D            5,R
11  | | | |H>(D>K)      1,R
12  | | | |D>K          9,11,>E
13  | | | |K            10,12,>E
14  | | | | |M          A
15  | | | | |K          13,R
16  | | | | |K&M        14,15,&I
17  | | | | |(K&M)>P    2,R
18  | | | | |P          16,17,>E
19  | | | |M>P          14-18,>I
20  | | |-I             6-19,-I
21  | |D>-I             5-20,>I
22  |H>(D>-I)            4-21,>I
```

6-4.

```
|X|Y          |X|Y          |-X
|                           |-Y
|-X  |E       |-Y  |E       |X|Y  |I
```


7-1. a)

```
1   |AvB                 P
2   | |A                 A
3   | |BvA               2,vI
4   | |B                 A
5   | |BvA               4,vI
6   |BvA                 1,2-3,4-5,AC
```

7-1. b)

```
1   |Av(BvC)             P
2   | |A                 A
3   | |AvB               2,vI
4   | |(AvB)vC           3,vI
5   | |BvC               A
6   | | |B               A
7   | | |AvB             6,vI
8   | | |(AvB)vC         7,vI
9   | | |C               A
10  | | |(AvB)vC         9,vI
11  | |(AvB)vC           5,6-8,9-10,AC
12  |(AvB)vC             1,2-4,5-11,AC
```

7-1. c)

```
1   |(AvB)&(B>C)         P
2   |AvB                 1,&E
3   |B>C                 1,&E
4   | |A                 A
5   | |AvC               4,vI
6   | |B                 A
7   | |B>C               3,R
8   | |C                 6,7,>E
9   | |AvC               8,vI
10  |AvC                 2,4-5,6-9,AC
```

7-1. d)

```
1   |(A&B)v(A&C)         P
2   | |A&B               A
3   | |A                 2,&E
4   | |B                 2,&E
5   | |BvC               4,vI
6   | |A&(BvC)           3,5,&I
7   | |A&C               A
8   | |A                 7,&E
9   | |C                 7,&E
10  | |BvC               9,vI
11  | |A&(BvC)           8,10,&I
12  |A&(BvC)             1,2-6,7-11,AC
```

7-1. e)

```
1   |A&(BvC)             P
2   |A                   1,&E
3   |BvC                 1,&E
4   | |B                 A
5   | |A                 2,R
6   | |A&B               4,5,&I
7   | |(A&B)v(A&C)       6,vI
8   | |C                 A
9   | |A                 2,R
10  | |A&C               8,9,&I
11  | |(A&B)v(A&C)       10,vI
12  |(A&B)v(A&C)         3,4-7,8-11,AC
```

7-1. f)

```
1  | Av(B&C)           P
   |--------
2  | | A               A
   | |------
3  | | AvB             2,vI
4  | | AvC             2,vI
5  | | (AvB)&(AvC)     3,4,&I

6  | | B&C             A
   | |------
7  | | B               6,&E
8  | | C               6,&E
9  | | AvB             7,vI
10 | | AvC             8,vI
11 | | (AvB)&(AvC)     9,10,&I
12 | (AvB)&(AvC)       1,2-5,6-11,AC
```

7-1. g)

```
1  | (AvB)&(AvC)       P
   |--------
2  | AvB               1,&E
3  | AvC               1,&E
4  | | A               A
   | |------
5  | | Av(B&C)         4,vI

6  | | B               A
   | |------
7  | | AvC             3,R
8  | | | A             A
   | | |----
9  | | | Av(B&C)       8,vI

10 | | | C             A
   | | |----
11 | | | B             6,R
12 | | | B&C           10,11,&I
13 | | | Av(B&C)       12,vI
14 | | Av(B&C)         7,8-9,10-13,AC
15 | Av(B&C)           2,4-5,6-14,AC
```

7-1. h)

```
1  | KvL               P
2  | K<->L             P
   |--------
3  | | K               A
   | |------
4  | | K<->L           2,R
5  | | K>L             4,<>E
6  | | L               3,5,>E
7  | | K&L             3,6,&I

8  | | L               A
   | |------
9  | | K<->L           2,R
10 | | L>K             9,<>E
11 | | K               8,10,>E
12 | | K&L             8,11,&I

13 | K&L               1,3-7,8-12,AC
```

7-1. i)

```
1  | (D>G)v(D>I)       P
   |--------
2  | | D>G             A
   | |------
3  | | | D             A
   | | |----
4  | | | D>G           2,R
5  | | | G             3,4,>E
6  | | | GvI           5,vI
7  | | D>(GvI)         3-6,>I

8  | | D>I             A
   | |------
9  | | | D             A
   | | |----
10 | | | D>I           8,R
11 | | | I             9,10,>E
12 | | | GvI           11,vI
13 | | D>(GvI)         9-12,>I
14 | D>(GvI)           1,2-7,8-13,AC
```


7-1. j)

```
1  | -CvK              P
2  | A>D               P
   |--------
3  | | -C              A
   | |------
4  | | | AvC           A
   | | |----
5  | | | -C            3,R
6  | | | A             4,5,vE
7  | | | A>D           2,R
8  | | | D             6,7,>E
9  | | | KvD           8,vI
10 | | (AvC)>(KvD)     4-9,>I

11 | | K               A
   | |------
12 | | | AvC           A
   | | |----
13 | | | K             11,R
14 | | | KvD           13,vI
15 | | (AvC)>(KvD)     12-14,>I
16 | (AvC)>(KvD)       1,3-10,11-15,AC
```

7-1. k)

```
1  | -HvM              P
2  | -M>-C             P
   |--------
3  | | HvC             A
   | |------
4  | | | H             A
   | | |----
5  | | | -HvM          1,R
6  | | | | -H          A
   | | | |--
7  | | | | H           4,R
8  | | | --H           6-7,-I
9  | | | M             5,8,vE

10 | | | C             A
   | | |----
11 | | | | -M          A
   | | | |--
12 | | | | -M>-C       2,R
13 | | | | -C          11,12,>E
14 | | | | C           10,R
15 | | | --M           11-14,-I
16 | | | M             15,-E
17 | | M               3,4-9,10-16,AC
18 | (HvC)>M           3-17,>I
```

7-1. l)

```
1  | (S&J)v(-S&-J)     P
   |--------
2  | | S&J             A
   | |------
3  | | | S             A
   | | |----
4  | | | S&J           2,R
5  | | | J             4,&E
6  | | S>J             3-5,>I
7  | | | J             A
   | | |----
8  | | | S&J           2,R
9  | | | S             8,&E
10 | | J>S             7-9,>I
11 | | S<->J           6,10,<>I

12 | | -S&-J           A
   | |------
13 | | | S             A
   | | |----
14 | | | | -J          A
   | | | |--
15 | | | | -S&-J       12,R
16 | | | | -S          15,&E
17 | | | | S           13,R
18 | | | --J           14-17,-I
19 | | | J             18,-E
20 | | S>J             13-19,>I
21 | | | J             A
   | | |----
22 | | | | -S          A
   | | | |--
23 | | | | -S&-J       12,R
24 | | | | -J          23,&E
25 | | | | J           21,R
26 | | | --S           22-25,-I
27 | | | S             26,-E
28 | | J>S             21-27,>I
29 | | S<->J           20,28,<>I
30 | S<->J             1,2-11,12-29,AC
```

7-1. m)

```
1   |K>(FvC)            P
2   |J>(CvD)            P
3   |-C                 P
    |-----------------
4   | |-(FvD)           A
    | |---------------
5   | | |KvJ           A
    | | |-------------
6   | | | |K           A
    | | | |-----------
7   | | | |K>(FvC)     1,R
8   | | | |FvC         6,7,>E
9   | | | |-C          3,R
10  | | | |F           8,9,vE
11  | | | |FvD         10,vI
    | | |
12  | | | |J           A
    | | | |-----------
13  | | | |J>(CvD)     2,R
14  | | | |CvD         12,13,>E
15  | | | |-C          3,R
16  | | | |D           14,15,vE
17  | | | |FvD         16,vI
18  | | |FvD      5,6-11,12-17,AC
19  | | |-(FvD)         4,R
20  | |-(KvJ)           5-19,-I
21  |-(FvD)>-(KvJ)      4-20,>I
```

7-2. We have to show that, making free use of all rules except vE, if we i addition assume vE, then we can prove AC, and if we assume AC we can prove vE. The first half of this was done in the text. To show the second hal suppose we have a derivation on which 'XvY' and '-X' already appear. We r to derive Y, using AC and any other rules except vE.

```
1   |XvY              P
2   |-X               P
    |---------------
3   | |X              A
    | |-------------
4   | | |-Y           A
    | | |-----------
5   | | |X            3,R
6   | | |-X           2,R
7   | |--Y            4-6,-I
8   | |Y              7,-E
    |
9   | |Y              A
    | |-------------
10  | |Y              9,R
11  |Y                1,3-8,9-10,AC
```


7-3. a) <->I

```
1   | |X            A     —
    | |-----------          |
2   | |Y                    |Input for derived rule
    |                       |
3   | |Y            A       |
    | |-----------          |
4   | |X                  —
    |
5   |X>Y            1,2,>I
6   |Y>X            3,4,>I
7   |X<->Y            5,6,<>I
```

7-3. b) <->E

```
1   |X<->Y          —
2   |X              —   |Input for derived rule
    |
3   |X>Y            1,<>E
4   |Y              2,3,>E
```

7-3. c) <->E

```
1   |X<->Y          —
2   |Y              —   |Input for derived rule
    |
3   |Y>X            1,<>E
4   |X              2,3,>E
```

7-3. d) vE

```
1   |-XvY           —
2   |X              —   |Input for derived rule
    |
3   | |-Y           A
    | |-----------
4   | |-XvY         1,R
5   | |-X           3,4,vE
6   | |X            2,R
7   |--Y            3-6,-I
8   |Y              7,-E
```

7-3. e) vE

```
1  |Xv-Y                   |Input for derived rule
2  |Y
   |
3  | |-X              A
   | |---------------------
4  | |Xv-Y            1,R
5  | |-Y              3,4,vE
6  | |Y               2,R
7  |--X               3-6,-I
8  |X                 7,-E
```

7-3. f) DC

```
1  |X>Y                    |Input for derived rule
2  |-Y
   |
3  | |X               A
   | |---------------------
4  | |X>Y             1,R
5  | |Y               3,4,>E
6  | |-Y              2,R
7  |-X                3-6,-I
```

7-3. g) DC

```
1  |X>-Y                   |Input for derived rule
2  |Y
   |
3  | |X               A
   | |---------------------
4  | |X>-Y            1,R
5  | |-Y              3,4,>E
6  | |Y               2,R
7  |-X                3-6,-I
```

7-3. h) RD

```
1  | |-X              A       |
   | |---------------------   |Input for derived rule
2  | |Y                       |
3  | |-Y                      |
   |
4  |--X               1-3,-I
5  |X                 4,-E
```


7-3. i) DM

```
1  |-(XvY)          Input for derived
   |                rule
2  | |X               A
   | |---------------------
3  | |XvY             2,vI
4  | |-(XvY)          1,R
5  |-X                2-4,-I
6  | |Y               A
   | |---------------------
7  | |XvY             6,vI
8  | |-(XvY)          1,R
9  |-Y                6-8,-I
10 |-X&-Y             5,9,&I
```

7-3. j) DM

```
1  |-X&-Y             Input for
   |                  derived rule
2  | |XvY             A
   | |---------------------
3  | |-X&-Y           1,R
4  | |-X              3,&E
5  | |Y               2,4,vE
6  | |-Y              3,&E
7  |-(XvY)            2-6,-I
```

7-3. k) DM

```
1  |-(X&Y)            Input for
   |                  derived rule
2  | |-(-Xv-Y)        A
   | |---------------------
3  | |--X&--Y         2,DM
4  | |--X             3,&E
5  | |--Y             3,&E
6  | |X               4,-E
7  | |Y               5,-E
8  | |X&Y             6,7,&I
9  | |-(X&Y)          1,R
10 |--(-Xv-Y)         2-9,-I
11 |-Xv-Y             10,-E
```

7-3. l) DM

```
1  |-Xv-Y             Input for
   |                  derived rule
2  | |X&Y             A
   | |---------------------
3  | |X               2,&E
4  | |Y               2,&E
5  | |-Xv-Y           1,R
6  | |-Y              3,5,vE
7  |-(X&Y)            2-6,-I
```

7-3. m) CP

```
1  |X>Y               Input for
   |                  derived rule
2  | |-Y              A
   | |---------------------
3  | | |X             A
   | | |-------------------
4  | | |X>Y           1,R
5  | | |Y             3,4,>E
6  | | |-Y            2,R
7  | |-X              3-6,-I
8  |-Y>-X             2-7,>I
```

7-3. n) CP

```
1  |-X>Y              Input for
   |                  derived rule
2  | |-Y              A
   | |---------------------
3  | | |-X            A
   | | |-------------------
4  | | |-X>Y          1,R
5  | | |Y             3,4,>E
6  | | |-Y            2,R
7  | |--X             3-6,-I
8  | |X               7,-E
9  |-Y>X              2-8,>I
```

7-3. o) CP

```
1   | X>-Y              Input for
    |                   derived rule
2   | | Y               A
    | |-------
3   | | | X             A
    | | |-------
4   | | | X>-Y          1,R
5   | | | -Y            3,4,>E
6   | | | Y             2,R
7   | | -X              3-6,-I
8   | Y>-X              2-7,>I
```

7-3. p) C

```
1   | X>Y               Input for
    |                   derived rule
2   | | -(-XvY)         A
    | |-------
3   | | --X&-Y          2,DM
4   | | --X             3,&E
5   | | -Y              3,&E
6   | | X               4,-E
7   | | X>Y             1,R
8   | | Y               6,7,>E
9   | --(-XvY)          2-8,-I
10  | -XvY              9,-E
```

7-3. q) C

```
1   | -XvY              Input for
    |                   derived rule
2   | | X               A
    | |-------
3   | | -XvY            1,R
4   | | | -X            A
    | | |-------
5   | | | X             2,R
6   | | --X             4-5,-I
7   | | Y               3,6,vE
8   | X>Y               2-7,>I
```

7-3. r) C

```
1   | -(X>Y)            Input for
    |                   derived rule
2   | | -X              A
    | |-------
3   | | -Y>-X           2,W
4   | | X>Y             3,CP
5   | | -(X>Y)          1,R
6   | X                 2-5,RD
7   | | Y               A
    | |-------
8   | | X>Y             7,W
9   | | -(X>Y)          1,R
10  | -Y                7-9,-I
11  | X&-Y              6,10,&I
```

7-3. s) C

```
1   | X&-Y              Input for
    |                   derived rule
2   | | X>Y             A
    | |-------
3   | | X&-Y            1,R
4   | | X               3,&E
5   | | Y               2,4,>E
6   | | -Y              3,&E
7   | -(X>Y)            2-6,-I
```

7-4. a)

```
1   | M&(-BvC)          P
    |-------
2   | -BvC              1,&E
3   | B>C               2,C
```

7-4. b)

```
1   | M>(D>P)           P
2   | M>D               P
    |-------
3   | | M               A
    | |-------
4   | | M>D             2,R
5   | | D               3,4,>E
6   | | M>(D>P)         1,R
7   | | D>P             3,6,>E
8   | | P               5,7,>E
9   | M>P               3-8,>I
```


7-4. c)

```
1   | (K>S)>(S>H)       P
2   | S                 P
    |-------
3   | K>S               2,W
4   | S>H               1,3,>E
5   | H                 2,4,>E
6   | K>H               5,W
```

7-4. d)

```
1   | F>O               P
2   | L>J               P
    |-------
3   | | FvL             A
    | |-------
4   | | | F             A
    | | |-------
5   | | | F>O           1,R
6   | | | O             4,5,>E
7   | | | OvJ           6,vI
8   | | | L             A
    | | |-------
9   | | | L>J           2,R
10  | | | J             8,9,>E
11  | | | OvJ           10,vI
12  | | OvJ             3,4-7,8-11,AC
13  | (FvL)>(OvJ)       3-14,>I
```

7-4. e)

```
1   | -((F&H)v-F)       P
    |-------
2   | -(F&H)&--F        1,DM
3   | -(F&H)            2,&E
4   | --F               2,&E
5   | -Fv-H             3,DM
6   | -H                4,5,vE
```

7-4. f)

```
1   | B>(HvR)           P
    |-------
2   | | B               A
    | |-------
3   | | B>(HvR)         1,R
4   | | HvR             2,3,>E
5   | | | -H            A
    | | |-------
6   | | | HvR           4,R
7   | | | R             5,6,vE
8   | | -H>R            5-7,>I
9   | B>(-H>R)          2-8,>I
```

7-4. g)

```
1   | -(-M&D)           P
2   | F>-M              P
3   | Fv-D              P
    |-------
4   | --Mv-D            1,DM
5   | | --M             A
    | |-------
6   | | F>-M            2,R
7   | | -F              5,6,DC
8   | | Fv-D            3,R
9   | | -D              7,8,vE
10  | | -D              A
    | |-------
11  | | -D              10,R
12  | -D                4,5-9,10-11,AC
```

7-4. h)

```
1   | (A>B)&(D>-B)      P
2   | (CvD)&(C>-B)      P
    |-------
3   | | A               A
    | |-------
4   | | (A>B)&(D>-B)    1,R
5   | | A>B             4,&E
6   | | B               3,5,>E
7   | | D>-B            4,&E
8   | | -D              6,7,DC
9   | | (CvD)&(C>-B)    2,R
10  | | CvD             9,&E
11  | | C               8,10,vE
12  | | C>-B            9,&E
13  | | -B              11,12,>E
14  | -A                3-13,-I
```

7-4. i)

```
1   | P>(DvM)             P
    |---------------
2   | -Pv(DvM)            1,C
3   | | -P                A
    | |-------------
4   | | -PvD              3,vI
5   | | P>D               4,C
6   | | (P>D)v(P>M)       5,vI
    |
7   | | DvM               A
    | |-------------
8   | | | D               A
    | | |-----------
9   | | | P>D             8,W
10  | | | (P>D)v(P>M)     9,vI
    | |
11  | | | M               A
    | | |-----------
12  | | | P>M             11,W
13  | | | (P>D)v(P>M)     12,vI
14  | | (P>D)v(P>M)  7,8-10,11-13,AC
15  | (P>D)v(P>M)         2,3-6,7-14,AC
```

7-4. j)

```
1   | (G&-M)>(-M&K)       P
2   | K>-G                P
    |---------------
3   | | G                 A
    | |-------------
4   | | | -M              A
    | | |-----------
5   | | | G               3,R
6   | | | G&-M            4,5,&I
7   | | | (G&-M)>(-M&K)   1,R
8   | | | -M&K            6,7,>E
9   | | | K               8,&E
10  | | | K>-G            2,R
11  | | | -G              9,10,>E
12  | | M                 4-11,RD
13  | G>M                 3-12,>I
```

7-4. k)

```
1   | AvB                 P
2   | -B<->(CvD)          P
3   | (D&E)v(D&(F>G))     P
    |---------------
4   | | -A                A
    | |-------------
5   | | AvB               1,R
6   | | B                 4,5,vE
7   | | -B<->(CvD)        2,R
8   | | (CvD)>-B          7,<>E
9   | | -(CvD)            6,8,DC
10  | | -C&-D             9,DM
11  | | -D                10,&E
12  | | (D&E)v(D&(F>G))   3,R
13  | | | D&E             A
    | | |-----------
14  | | | D               13,&E
    | |
15  | | | D&(F>G)         A
    | | |-----------
16  | | | D               15,&E
17  | | D            12,13-14,15-16,AC
18  | A                   4-17,RD
```

7-4. l)

```
1   | S<->J               P
    |---------------
2   | S>J                 1,<>E
3   | J>S                 1,<>E
4   | -SvJ                2,C
5   | -JvS                3,C
6   | | -S                A
    | |-------------
7   | | -JvS              5,R
8   | | -J                6,7,vE
9   | | -S&-J             6,8,&I
10  | | (S&J)v(-S&-J)     9,vI
    |
11  | | J                 A
    | |-------------
12  | | -JvS              5,R
13  | | S                 11,12,vE
14  | | S&J               11,13,&I
15  | | (S&J)v(-S&-J)     14,vI
16  | (S&J)v(-S&-J)  4,6-10,11-15,AC
```


7-4. m)

```
1   | -C>(Fv-(DvN))       P
2   | -N>D                P
    |---------------
3   | | -F                A
    | |-------------
4   | | | -C              A
    | | |-----------
5   | | | -C>(Fv-(DvN))   1,R
6   | | | Fv-(DvN)        4,5,>E
7   | | | -F              3,R
8   | | | -(DvN)          6,7,vE
9   | | | -D&-N           8,DM
10  | | | -D              9,&E
11  | | | -N              9,&E
12  | | | -N>D            2,R
13  | | | D               11,12,>E
14  | | C                 4-13,RD
15  | -F>C                3-14,>I
```

7-4. n)

```
1   | (GvA)>(H>B)         P
2   | (H>(H&B))>K         P
    |---------------
3   | | G                 A
    | |-------------
4   | | GvA               3,vI
5   | | (GvA)>(H>B)       1,R
6   | | H>B               4,5,>E
7   | | | H               A
    | | |-----------
8   | | | H>B             6,R
9   | | | B               7,8,>E
10  | | | H&B             7,9,&I
11  | | H>(H&B)           7-10,>I
12  | | (H>(H&B))>K       2,R
13  | | K                 11,12,>E
14  | G>K                 3-13,>I
```

7-4. o)

```
1   | F>(KvB)             P
2   | (-FvG)&(-Gv-K)      P
    |---------------
3   | -FvG                2,&E
4   | -Gv-K               2,&E
5   | | F                 A
    | |-------------
6   | | F>(KvB)           1,R
7   | | KvB               5,6,>E
8   | | -FvG              3,R
9   | | G                 5,8,vE
10  | | -Gv-K             4,R
11  | | -K                9,10,vE
12  | | B                 7,11,vE
13  | F>B                 5-12,>I
```

7-4. p)

```
1   | Dv(M>J)             P
2   | (M>(M&J))>(PvK)     P
3   | (P>D)&(K>F)         P
    |---------------
4   | P>D                 3,&E
5   | K>F                 3,&E
6   | | D                 A
    | |-------------
7   | | DvF               6,vI
    |
8   | | M>J               A
    | |-------------
9   | | | M               A
    | | |-----------
10  | | | M>J             8,R
11  | | | J               9,10,>E
12  | | | M&J             9,11,&I
13  | | M>(M&J)           9-12,>I
14  | | (M>(M&J))>(PvK)   2,R
15  | | PvK               13,14,>E
16  | | | P               A
    | | |-----------
17  | | | P>D             4,R
18  | | | D               16,17,>E
19  | | | DvF             18,vI
    | |
20  | | | K               A
    | | |-----------
21  | | | K>F             5,R
22  | | | F               20,21,>E
23  | | | DvF             22,vI
24  | | DvF          15,16-19,20-23,AC
25  | DvF                 1,6-7,8-24,AC
```

7-4. q)

```
1   |Q<->-(A&F)          P
2   |-(MvA)>-H           P
3   |-(Q&A)vF            P
4   | |Q                 A
5   | | |H               A
6   | | |-(MvA)>-H       2,R
7   | | |--(MvA)         5,6,DC
8   | | |MvA             7,-E
9   | | |Q               4,R
10  | | |Q<->-(A&F)      1,R
11  | | |-(A&F)          9,10,<>E
12  | | |-Av-F           11,DM
13  | | | |-A           A
14  | | | |MvA          8,R
15  | | | |M            13,14,vE
16  | | | |-F           A
17  | | | |-(Q&A)vF     3,R
18  | | | |-(Q&A)       16,17,vE
19  | | | |-Qv-A        18,DM
20  | | | |Q            9,R
21  | | | |-A           19,20,vE
22  | | | |MvA          8,R
23  | | | |M            21,22,vE
24  | | |M               12,AC
25  | |H>M               5-24,>I
26  |Q>(H>M)             4-25,>I
```

7-4. r)

```
1   |(I&-T)>P            P
2   |-A>-T               P
3   |-TvC                P
4   |C>D                 P
5   | |-P                A
6   | | |I               A
7   | | |-P              5,R
8   | | |(I&-T)>P        1,R
9   | | |-(I&-T)         7,8,DC
10  | | |-Iv--T          9,DM
11  | | |--T             6,10,vE
12  | | |-A>-T           2,R
13  | | |--A             11,12,DC
14  | | |A               13,-E
15  | | |-TvC            3,R
16  | | |C               11,15,vE
17  | | |C>D             4,R
18  | | |D               16,17,>E
19  | | |D&A             14,18,&I
20  | |I>(D&A)           6-19,>I
21  |-P>(I>(D&A))        5-20,>I
```

7-4. s)

```
1   |B>(NvM)             P
2   |N>(C&K)             P
3   |C>(K>P)             P
4   |-(P&B)              P
5   | |B                 A
6   | |B>(NvM)           1,R
7   | |NvM               5,6,>E
8   | | |-M              A
9   | | |NvM             7,R
10  | | |N               8,9,vE
11  | | |N>(C&K)         2,R
12  | | |C&K             10,11,>E
13  | | |C               12,&E
14  | | |K               12,&E
15  | | |C>(K>P)         3,R
16  | | |K>P             13,15,>E
17  | | |P               14,16,>E
18  | | |B               5,R
19  | | |P&B             17,18,&I
20  | | |-(P&B)          4,R
21  | |M                 8-20,RD
22  |B>M                 5-21,>I
```


7-5. Suppose we have a derivation with X as its only premise and Y and -Y as conclusions. Relabel X as an assumption, and make the whole derivation the sub-derivation of a premiseless outer-derivation. The sub-derivation licenses drawing -X as final conclusion of its outer-derivation by applying RD. Thus any instance of the new test for contradiction can be converted to an instance of the old test.

7-6. a)

```
1   | |AvB               A
2   | | |-B              A
3   | | |AvB             1,R
4   | | |A               2,3,vE
5   | |-B>A              2-4,>I
6   |(AvB)>(-B>A)        1-5,>I
```

7-6. b)

```
1   | |-(Mv-(M&N))       A
2   | |-M&--(M&N)        1,DM
3   | |-M                2,&E
4   | |--(M&N)           2,&E
5   | |M&N               4,-E
6   | |M                 5,&E
7   |Mv-(M&N)            1-6,RD
```

7-6. c)

```
1   | |H>(O>N)           A
2   | | |H&O             A
3   | | |H               2,&E
4   | | |O               2,&E
5   | | |H>(O>N)         1,R
6   | | |O>N             3,5,>E
7   | | |N               4,6,>E
8   | |(H&O)>N           2-7,>I
9   |(H>(O>N))>((H&O)>N)    1-8,>I
```

7-6. d)

```
1   | |D>B               A
2   | | |D>T             A
3   | | | |D             A
4   | | | |D>T           2,R
5   | | | |T             3,4,>E
6   | | | |D>B           1,R
7   | | | |B             3,6,>E
8   | | | |B&T           5,7,&I
9   | | |D>(B&T)         3-8,>I
10  | |(D>T)>(D>(B&T))   2-9,>I
11  |(D>B)>((D>T)>(D>(B&T)))    1-10,>I
```

7-6. e)

```
1   | |K>F               A
2   | | |K&P             A
3   | | |K               2,&E
4   | | |K>F             1,R
5   | | |F               3,4,>E
6   | |(K&P)>F           2-5,>I
7   | |-F>-(K&P)         6,CP
8   |(K>F)>(-F>-(K&P))   1-7,>I
```

7-6. f)

```
1   | |(FvG)>(P&Q)       A
2   | | |F               A
3   | | |FvG             2,vI
4   | | |(FvG)>(P&Q)     1,R
5   | | |P&Q             3,4,>E
6   | | |Q               5,&E
7   | |F>Q               2-6,>I
8   | |-Q>-F             7,CP
9   |((FvG)>(P&Q))>(-Q>-F)   1-8,>I
```

7-6. g)

```
1  | L>(M>N)                 A
2  | | L>M                   A
3  | | | L                   A
4  | | | L>M                 2,R
5  | | | M                   3,4,>E
6  | | | L>(M>N)             1,R
7  | | | M>N                 3,6,>E
8  | | | N                   5,7,>E
9  | | L>N                   3-8,>I
10 | (L>M)>(L>N)             2-9,>I
11 | (L>(M>N))>((L>M)>(L>N))    1-10,>I
```

7-6. h)

```
1  | (SvT)>F                 A
2  | | (FvG)>H               A
3  | | | S                   A
4  | | | SvT                 3,vI
5  | | | (SvT)>F             1,R
6  | | | F                   4,5,>E
7  | | | FvG                 6,vI
8  | | | (FvG)>H             2,R
9  | | | H                   7,8,>E
10 | | S>H                   3-9,>I
11 | ((FvG)>H)>(S>H)         2-10,>I
12 | ((SvT)>F)>(((FvG)>H)>(S>H))
                             1-11,>I
```

7-6. i)

```
1  | -((I&-J)v((J&K)v-(K&I)))      A
2  | -(I&-J)&-((J&K)v-(K&I))       1,DM
3  | -(I&-J)                       2,&E
4  | -((J&K)v-(K&I))               2,&E
5  | -(J&K)&--(K&I)                4,DM
6  | --(K&I)                       5,&E
7  | K&I                           6,-E
8  | K                             7,&E
9  | I                             7,&E
10 | -Iv--J                        3,DM
11 | --J                           9,10,vE
12 | -(J&K)                        5,&E
13 | -Jv-K                         12,DM
14 | -K                            11,13,vE
15 | (I&-J)v((J&K)v-(K&I))         1-15,RD
```

7-6. j)

```
1  | -(((C&(AvD))v-(C&F))v-(A&-G))  A
2  | -((C&(AvD))v-(C&F))&--(A&-G)
   |                                1,DM
3  | -((C&(AvD))v-(C&F))            2,&E
4  | --(A&-G)                       2,&E
5  | A&-G                           4,-E
6  | A                              5,&E
7  | -(C&(AvD))&--(C&F)             3,DM
8  | -(C&(AvD))                     7,&E
9  | --(C&F)                        7,&E
10 | C&F                            9,-E
11 | C                              10,&E
12 | -Cv-(AvD)                      8,DM
13 | -(AvD)                         11,12,vE
14 | AvD                            6,vI
15 | ((C&(AvD))v-(C&F))v-(A&-G)
                                    1-15,RD
```

7-7. a)

```
1 | A&-A              P
2 | A                 1,&E
3 | -A                1,&E
```

7-7. b)

```
1 | (Hv-B)&((-B>H)&-H)    P
2 | Hv-B                  1,&E
3 | (-B>H)&-H             1,&E
4 | -B>H                  3,&E
5 | -H                    3,&E
6 | B                     4,5,DC
7 | H                     2,6,vE
```

7-7. c)

```
1  | ((H&F)>C)&-(H>(F>C))   P
2  | (H&F)>C                1,&E
3  | -(H>(F>C))             1,&E
4  | H&-(F>C)               3,C
5  | H                      4,&E
6  | -(F>C)                 4,&E
7  | F&-C                   6,C
8  | F                      7,&E
9  | -C                     7,&E
10 | H&F                    5,8,&I
11 | C                      2,10,>E
```


7-7. d)

```
1  | (-(GvQ)&(K>G))&-(Pv-K)    P
2  | -(GvQ)&(K>G)              1,&E
3  | -(Pv-K)                   1,&E
4  | -(GvQ)                    2,&E
5  | K>G                       2,&E
6  | -G&-Q                     4,DM
7  | -G                        6,&E
8  | -K                        5,7,DC
9  | -P&--K                    3,DM
10 | --K                       9,&E
```

7-7. e)

```
1  | (K>(D>P))&((-KvD)&-(K>P))   P
2  | K>(D>P)                     1,&E
3  | (-KvD)&-(K>P)               1,&E
4  | -KvD                        3,&E
5  | -(K>P)                      3,&E
6  | K&-P                        5,C
7  | K                           6,&E
8  | -P                          6,&E
9  | D>P                         2,7,>E
10 | D                           4,7,vE
11 | P                           9,10,>E
```

7-7. f)

```
1  | -(-(Nv-R)>(N<->-R))     P
2  | -(Nv-R)&-(N<->-R)       1,C
3  | -(Nv-R)                 2,&E
4  | -(N<->-R)               2,&E
5  | -N&--R                  3,DM
6  | -N                      5,&E
7  | --R                     5,&E
8  | R                       7,-E
9  | | -N                    A
10 | | R                     8,R
11 | -N>R                    9-10,>I
12 | -R>N                    11,CP
13 | | R                     A
14 | | -N                    6,R
15 | R>-N                    13-14,>I
16 | N>-R                    15,CP
17 | N<->-R                  12,16,<>I
```

7-7. g)

```
1  | (FvG)<->(-F&-G)      P
2  | | -F&-G              A
3  | | (FvG)<->(-F&-G)    1,R
4  | | FvG                2,3,<>E
5  | | -(FvG)             2,DM
6  | -(-F&-G)             2-5,-I
7  | | FvG                A
8  | | (FvG)<->(-F&-G)    1,R
9  | | -F&-G              7,8,<>E
10 | | -(FvG)             9,DM
11 | -(FvG)               7-10,-I
12 | -F&-G                11,DM
```

7-7. h)

```
1  | (-(FvG)v(P&Q))&-(-Q>-F)    P
2  | -(FvG)v(P&Q)               1,&E
3  | -(-Q>-F)                   1,&E
4  | -Q&--F                     3,C
5  | -Q                         4,&E
6  | --F                        4,&E
7  | F                          6,-E
8  | FvG                        7,vI
9  | P&Q                        2,8,vE
10 | Q                          9,&E
```

7-7. i)

```
1  | (A>D)&(((A&-B)v(A&-C))&((B&-D)v(B&C)))    P
   |---
2  | A>D                          1,&E
3  | ((A&-B)v(A&-C))&((B&-D)v(B&C))   1,&E
4  | (A&-B)v(A&-C)                3,&E
5  | (B&-D)v(B&C)                 3,&E
6  | A&(-Bv-C)                    4,DS
7  | B&(-DvC)                     5,DS
8  | A                            6,&E
9  | -Bv-C                        6,&E
10 | B                            7,&E
11 | -DvC                         7,&E
12 | -C                           9,10,vE
13 | -D                           11,12,vE
14 | D                            2,8,>E
```

I've cheated in this problem-using the derived rule for the distributive law (DS), which I did not introduce.You have really done the work of proving the distributive law as a derived rule in problem 7-1d.

7-7. j)

```
1  | (A<->B)<->(-A<->B)           P
   |---
2  |  | A&B                       A
   |  |---
3  |  | A                         2,&E
4  |  | B                         2,&E
5  |  | B>A                       3,W
6  |  | A>B                       4,W
7  |  | A<->B                       5,6,<>I
8  |  | (A<->B)<->(-A<->B)     1,R
9  |  | -A<->B                      7,8,<>E
10 |  | -A                        4,9,<>E
11 | -(A&B)                       2-10,-I
12 |  | A&-B                      A
   |  |---
13 |  | A                         12,&E
14 |  | -B                        12,&E
15 |  | A>-B                      14,W
16 |  | B>-A                      15,CP
17 |  | -B>A                      13,W
18 |  | -A>B                      17,CP
19 |  | -A<->B                      16,18,<>I
20 |  | (A<->B)<->(-A<->B)     1,R
21 |  | A<->B                     19,20,<>E
22 |  | B                         13,21,<>E
23 | -(A&-B)                      12-22,-I
24 | -Av-B                        11,DM
25 | -Av--B                       23,DM
26 |  | A                         A
   |  |---
27 |  | -Av-B                     24,R
28 |  | -Av--B                    25,R
29 |  | -B                        26,27,vE
30 |  | --B                       26,28,vE
31 | -A                           26-30,-I
32 |  | -B                        A
   |  |---
33 |  | -A                        31,R
34 |  | -B>-A                     33,W
35 |  | A>B                       34,CP
36 |  | -A>-B                     32,W
37 |  | B>A                       36,CP
38 |  | A<->B                       35,37,<>I
39 |  | (A<->B)<->(-A<->B)     1,R
40 |  | -A<->B                      38,39,<>E
41 |  | B                         33,40,<>E
42 | B                            32-41,RD
43 | -A>B                         42,W
44 | B>-A                         31,W
45 | -A<->B                         43,44,<>I
46 | A<->B                          1,45,<>E
47 | A                            42,46,<>E
```


7-8. a) A sentence is a contradiction if and only if there is not an assignment of truth values to sentence letters which makes it true. Hence a set of sentences is inconsistent if and only if its conjunction is a contradiction. This characterization works only for finite sets of sentences, since there cannot be a conjunction of an infinite set of sentences.

b) A finite set of sentences is shown to be inconsistent if there is a derivation which has the sentences in the set as premises and contradicting conclusions.

c1)

```
1  | C<->G                 P
2  | G<->-C                P
   |---
3  |  | C                  A
   |  |---
4  |  | C<->G                1,R
5  |  | G                  3,4,<>E
6  |  | G<->-C               2,R
7  |  | -C                 5,6,<>E
8  | -C                    3-7,-I
9  |  | -C                 A
   |  |---
10 |  | G<->-C               2,R
11 |  | G                  9,10,<>E
12 |  | C<->G                1,R
13 |  | C                  11,12,<>E
14 | C                     9-13,RD
```

c2)

```
1 | FvT                   P
2 | (FvT)>(-F&-T)         P
  |---
3 | -F&-T                 1,2,>E
4 | -F                    3,&E
5 | -T                    3,&E
6 | T                     1,4,vE
```

c3)

```
1  | JvK                   P
2  | -Jv-K                 P
3  | J<->K                   P
   |---
4  |  | J                  A
   |  |---
5  |  | J<->K                3,R
6  |  | -Jv-K              2,R
7  |  | K                  4,5,<>E
8  |  | -J                 6,7,vE
9  |  | J&-J               4,8,&I
10 |  | K                  A
   |  |---
11 |  | -Jv-K              2,R
12 |  | -J                 10,11,vE
13 |  | J<->K                3,R
14 |  | J                  10,13,<>E
15 |  | J&-J               12,14,&I
16 | J&-J                  1,4-9,10-15,AC
17 | J                     16,&E
18 | -J                    16,&E
```

c4)

```
1 | (GvK)>A               P
2 | (AvH)>G               P
3 | G&-A                  P
  |---
4 | G                     3,&E
5 | -A                    3,&E
6 | GvK                   4,vI
7 | A                     1,6,>E
```

c5)

```
1   |D<->(-P&-M)       P
2   |P<->(J&-F)        P
3   |-Fv-D             P
4   |D&J               P
    |---------------------
5   |D                 4,&E
6   |J                 4,&E
7   |-F                3,5,vE
8   |J&-F              6,7,&I
9   |P                 2,8,<>E
10  |-P&-M             1,5,<>E
11  |-P                10,&E
```


8-1. a)

```
1           D            P
2           J            P
3      * -(D v J)        -C        VALID
4          -D            3 -v
5          -J            3 -v
            X
```

8-1. b)

```
1      * -(M v -N)       P
2           -N           -C
3           -M           1 -v      VALID
4         * --N          1 -v
5            N           4 --
             X
```

8-1. c)

```
1      * -(F v P)        P
2     * -(-F v -P)       -C
3           -F           1 -v
4           -P           1 -v      VALID
5         * --F          2 -v
6          --P           2 -v
7            F           5 --
             X
```

8-1. d)

```
1                   * H v I                              P
2                  * -I v J                              P
3                  * -J v K                              P
4                 * -(H v K)                             -C
5                     -H                                 4 -v
6                     -K                                 4 -v
            |----------^----------|
7           H                     I                      1 v
            X            |-------^-------|
8                       -I               J               2 v
                         X           |---^---|
9                                   -J       K           3 v
                                     X       X
     VALID
```

8-2. Since a conjunction, X&Y, is true whenever both X and Y are true, when we have a conjunction on a tree, both conjuncts should be written on one branch directly below the conjunction:

```
X&Y
 X
 Y
```

For a negated conjunction, -(X&Y), we first use DeMorgan's Law to get -Xv-Y. Then, we simply use the rule for disjunctions.

Similarly, to get the rule for conditionals, X->Y, we use the Conditional Law to get -XvY, and then use the rule for disjunctions. Again, for negated conditionals, -(X->Y), we use the Conditional Law to get X&-Y, and then the rule for conjunctions.

Biconditionals are a bit more tricky. First, we consider what the biconditional X<->Y is logically equivalent to: (X&Y)v(-X&-Y). Since this is itself a disjunction, we will have two branches, each of which will have the decomposition products from a conjunction on it. the result is:

```
     X<->Y
X           -X
Y           -Y
```

For negated biconditionals, we note that -(X<->Y) is logically equivalent to -[(X->Y)&(Y->X)]. By DeMorgan's Law, this is equivalent to -(X->Y)v-(Y->X); and, by the conditional law, we get: (X&-Y)v(Y&-X). Again, we use the rules for disjunctions and conjunctions to get two branches, similar to that above.

8-3. a)

```
1                      F                          P
2                * -(F & K)                       -C
       |-------------^-------------|
3     -F                          -K              2 -&
       X
```

INVALID. C.E.: (F&-K)

8-3. b)

```
1               * -(-S & T)                       P
2                    -S                           -C
       |-------------^-------------|
3   * --S                         -T              1 -&
4      S                                          3 --
       X
```

INVALID. C.E.: (-S&-T)

8-3. c)

```
1                 * K v H                         P
2                    -K                           P
3               * -(H & D)                        -C
       |-------------^-------------|
4      K                           H              1 v
       X                    |------^------|
5                          -H            -D       3 -&
                            X
```

INVALID. C.E.: (-K&H&-D)

8-3. d)

```
1                        * -(I & P)                          P
2                          * P v F                           P
3                        * -(I v F)                          -C
4                            -I                              3 -v
5                            -F                              3 -v
            |-----------------^-----------------|
6          -I                                  -P            1 -&
     |------^------|                    |------^------|
7    P             F                    P             F      2 v
                   X                    X             X
```

INVALID. C.E.: (-I&-F&P)

8-4. a)

```
1                          A                          P
2                          B                          P
3                      * -(A & B)                     -C
     |---------------------^---------------------|
4   -A                                          -B    3 -&
     X                                           X
```

VALID

8-4. b)

```
1                     * -C > H                        P
2                    * -(-H > C)                      -C
3                        -H                           2 ->
4                        -C                           2 ->
     |-------------------^-------------------|
5  * --C                                     H        1 ->
6      C                                     X        5 --
       X
```

VALID

8-4. c)

```
1                        * K > F                      P
2                           F                         P
3                          -K                         -C
     |---------------------^---------------------|
4   -K                                           F    1 ->
```

INVALID. C.E.: (F&-K)

8-4. d)

```
1                 * P > M                  P
2                 * M > B                  P
3                * -(P > B)                -C
4                    P                     3 ->
5                   -B                     3 ->
        |-------------^-------------|
6       -P                          M      1 >
        X                  |--------^--------|
7                          -M                B   2 >
                            X                X
```

VALID

8-4. e)

```
1                       * F v G                 P
2                      * -G v H                 P
3                     * -(F & H)                -C
             |-------------^-------------|
4            F                           G      1 v
      |------^------|            |-------^-----|
5     -G            H            -G            H    2 v
   |--^--|      |---^---|        X         |---^---|
6  -F    -H     -F      -H                 -F      -H     3 -&
   X            X       X                          X
```

INVALID. C.E.: (F&-G&-H); (-F&G&H)

8-4. f)

```
1                * J > D                 P
2                  K > D                 P
3                * J & K                 P
4                  -D                    -C
5                   J                    3 &
6                   K                    3 &
      |-------------^-------------|
7     -J                          D      1 >
      X                           X
```

VALID

8-4. g)

```
1                     (K v S)                P
2                   * -(K & H)               P
3                   * -(K > H)               -C
4                        K                   3->
5                       -H                   3->
          |--------------^--------------|
6         K                             S    1 v
   |------^-------|             |-------^-------|
7  -K             -H            -K              -H   2-&
   X                            X
```

INVALID. C.E.: (K&-H&S)

8-4. h)

```
1                         * J > D                 P
2                         * K > D                 P
3                         * J v K                 P
4                           -D                    -C
                |------------^------------|
5               -J                        D       1 ->
         |------^------|                  X
6        -K            D                          2 ->
     |---^---|         X
7    J       K                                    3 v
     X       X
```

VALID

8-4. i)

```
1                         * M <-> N                     P
2                      * -(M <-> -N)                    -C
             |--------------^---------------|
3            M                              -M          1 <>
4            N                              -N          1 <>
      |------^------|                |------^------|
5     M             -M               M             -M   2 -<>
6   * --N           -N             * --N           -N   2 -<>
7     N             X                N                  6 --
                                     X
```

INVALID. C.E.: (-M&-N)

8-4. j)

```
1                      * T <-> I                 P
2                       * I & A                  P
3                    * -(-T v -A)                -C
4                          I                     2 &
5                          A                     2 &
6                       * --T                    3 -v
7                       * --A                    3 -v
8                          T                     6 --
9                          A                     7 --
        |------------------^------------------|
10      T                                     -T    1 <>
11      I                                     -I    1 <>
                                              X
```

INVALID. C.E.: (I&A&T)

8-4. k)

```
1                      * -(F & -L)                     P
2                      * -(L & -C)                     P
3                     * -(F <-> L)                     -C
              |--------------^----------------|
4            -F                         *  --L         1 -&
5             |                              L         4 --
       |------^------|           |-----------^---------|
6     -L         * --C          -L              *  --C 2-&
    |--^--|          |           X                   |
7   F     -F         |                               | 3 -<>
8   -L     L         |                               | 3 -<>
9   X      X         C                               C 6 --
                 |---^---|                       |---^---|
10               F      -F                       F      -F  3 -<>
11              -L       L                      -L       L  3 -<>
                 X                               X
```

INVALID. C.E.: (-F&C&L)

8-4. l)

```
1                      * F <-> G                       P
2                      * G <-> H                       P
3                    * -(F <-> H)                      -C
             |---------------^-------------------|
4            F                                  -F     1 <>
5            G                                  -G     1 <>
      |------^------|                 |---------^--------|
6     G            -G                 G                 -G   2 <>
7     H            -H                 H                 -H   2 <>
  |---^---|         X                 X            |----^----|
8 F      -F                                        F        -F  3 -<>
9 -H      H                                       -H         H  3 -<>
  X       X              VALID                     X         X
```

8-4. m)

```
1                       * C > N                        P
2                     * -(I > C)                       P
3                     * -(N v -I)                      P
4                    * -(-N > -I)                      -C
5                          I                           2 ->
6                         -C                           2 ->
7                         -N                           3 -v
8                       * --I                          3 -v
9                         -N                           4 ->
10                      * --I                          4 ->
11                         I                           8 --
12                         I                           10 --
            |--------------^------------------|
13          -C                                N        1 >
                                              X
```

INVALID. C.E.: (I&-C&-N)

8-4. n)

```
1                      * -(Q > D)                      P
2                      * -(Q & -B)                     P
3                      * -(B & -R)                     P
4                      * -(D v -Q)                     -C
5                           Q                          1 ->
6                          -D                          1 ->
7                          -D                          4 -v
8                        * --Q                         4 -v
9                           Q                          8 --
         |------------------^-----------------|
10      -Q                               * --B         2 -&
11       X                                  B          10 --
                                   |--------^--------|
12                                -B            * --R  3 -&
13                                 X                R  12 --
```

INVALID. C.E.: (Q&-D&B&R)

8-4. o)

```
1                       ● R <-> -S                     P
2                      ● -(R <-> T)                    P
3                       ● -(R & -S)                    -C
            |-----------------^--------------------|
4           R                                     -R       1 <>
5          -S                                  ● --S       1 <>
     |------^------|                               |
6    R            -R                               |       2 -<>
7   -T             T                               |       2 -<>
 |---^---|         X                               |
8 -R   * --S                                       |       3 -&
9  X       S                                       |       8 --
10 X                                               S       5 --
                                         |---------^--------|
11                                       R                 -R    2 -<>
12                                      -T                  T    2 -<>
                                         X            |-----^-----|
13                                                   -R        --S  3 -&
```

INVALID. C.E.: (-R&S&T)

8-4. p)

```
1                      * O <-> -F                      P
2                     * -(F <-> K)                     P
3                      * -(O > K)                      -C
4                           O                          3 ->
5                          -K                          3 ->
          |-----------------^------------------|
6         O                                   -O       1 <>
7        -F                                  --F       1 <>
    |-----^------|                             X
8   F           -F                                     2 -<>
9  -K            K                                     2 -<>
    X            X
```

VALID

8-5.

In doing truth trees, we are concerned with the truth values of atomic sentences which are minimally sufficient to make the original sentence true. Thus, if a rule tells us to write:

```
X
-Y
```

we could just as well write the stack in the reverse order. This is because it is the truth values of the atomic sentences that we are concerned with, so order does not matter. [Notice that X and -Y on the same stack corresponds to (X&-Y), which is logically equivalent to (-Y&X).] Similarly, the order of branches does not matter, since XvY is logically equivalent to YvX.

8-6.

By using DeMorgan's Law, we can change a negated conjunction -(X&Y) to a disjunction -Xv-Y, and then decompose this using the rule for disjunctions. In this way, we can do away with the rule for negated conjunctions. Similarly, we can change any negated disjunction -(XvY) to a conjunction, -X&-Y, and then use the rule for conjunctions.

We can also do away with the rules for disjunctions and conjunctions, and use only the rule for negated disjunctions and negated conjunctions. In this case, whenever we had a conjunction, X&Y, on a tree, we would use DeMorgan's Law to get -(-Xv-Y), and use the rule for negated disjunctions. Likewise, any disjunction, XvY, could be converted to a negated conjunction, -(-X&-Y), using DeMorgan's Law.

8-7.

a) Since a conjunction X&Y&Z is true whenever X and Y and Z are all true, whenever we have a conjunction with three conjuncts in a tree, we simply write:

```
X
Y
Z
```

underneath the original conjunction.

Similarly, any disjunction XvYvZ is true whenever either X or Y or Z is true. So, on a tree, we write three branches under the disjunction:

```
          XvYvZ
|----------|----------|
X          Y          Z
```

b) In general, for a conjunction of any length, X&Y&...&Z, in a tree, we write all of the conjuncts in one branch directly below the original conjunction. For a disjunction of any length, XvYv...vZ, in a tree, we write one branch for each of the disjuncts below the original disjunction.

8-8.

By the truth table definition of the Sheffer Stroke, we saw that X|Y is logically equivalent to -(XvY); and, by DeMorgan's Law, it is logically equivalent to -X&-Y. So, in a truth tree, whenever we see X|Y, we write -X and -Y on one branch directly below it. We also saw that X|X is logically equivalent to -X; so, whenever X|X appears in a truth tree, we simply write -X directly below it.

ANSWERS TO EXERCISES IN VOLUME I, CHAPTER 9

9-1. a) &, b) ->, c) -, d) ->.

9-2. a)

```
1                          B                          P
2                * -[(B v C) & (B v D)]               -C
          |--------------------^--------------------|
3     * -(B v C)                         * -(B v D)   2 -&
4          -B                                 -B      3 -v
5          -C                                 -D      3 -v
           X                                  X
```

VALID

9-2. b)

```
1                      * D v F                        P
2                       K > -F                        P
3                 * -[-F > (D v K)]                   -C
4                         -F                          3 ->
5                     * -(D v K)                      3 ->
6                         -D                          5 -v
7                         -K                          5 -v
          |--------------------^--------------------|
8         D                                         F 1 v
          X                                         X
```

VALID

9-2. c)

```
1                    * N > (D > P)                    P
2                      * N > D                        P
3                    * -(N > P)                       -C
4                          N                          3 ->
5                         -P                          3 ->
    |----------------------^-------------|
6   -N                               * D > P          1 >
    X                  |-----------------^-----------|
7                     -N                             D  2 >
                       X                  |----^-----|
8                                        -D          P  6 >
                                          X          X
```

VALID

9-2. d)

```
 1                * (H & P) v (S & -J)                  P
 2                  * J > -(H & D)                      P
 3                   * -(J > -D)                        -C
 4                        J                             3 ->
 5                      * --D                           3 ->
 6                        D                             5 --
                |--------------------|
 7        * H & P                  * S & -J             1 v
 8            H                        S                7 &
 9            P                        -J               7 &
     |--------------|                  X
10   -J        * -(H & D)                               2 >
     X          |---------|
11              -H        -D                            10 -&
                X         X           VALID
```

9-2. e)

```
 1                * (R v L) > (G v A)                   P
 2                    * R > -G                          P
 3                   * -(B & -R)                        P
 4                   * -(-B v A)                        -C
 5                      * --B                           4 -v
 6                        -A                            4 -v
 7                        B                             5 --
             |------------------------|
 8      * -(R v L)                 * G v A              1 >
 9          -R                                          8 -v
10          -L                                          8 -v
     |-----------|             |-----------|
11   -R          -G            -R          -G           2 >
  |-----|      |-----|      |-----|      |-----|
12 -B  * --R   -B  * --R    -B  * --R    -B  * --R      3 -&
13 X     R     X     R      X     R      X     R        12 --
         X           X            X           |--|
14                                            G  A      8 v
       VALID                                  X  X
```

9-2. f)

```
 1                 * (-I & -D) v -D                     P
 2              * --[(I & -J) v (D & -J)]               -C
 3                * (I & -J) v (D & -J)                 2 --
             |-------------------------|
 4       * -I & -D                     -D               1 v
 5           -I                                         4 &
 6           -D                                         4 &
      |--------------|             |--------|
 7  * I & -J      * D & -J      * I & -J   * D & -J     3 v
 8     I              D             I          D        7 &
 9     -J             -J            -J         -J       7 &
       X              X                        X
```

INVALID. C.E.: (-D&I&-J)

9-2. g)

```
 1                   * (G > B) & (-G > N)               P
 2                        * B v -N                      P
 3                      * -(N v -B)                     -C
 4                        * G > B                       1 &
 5                        * -G > N                      1 &
 6                           -N                         3 -v
 7                         * --B                        3 -v
 8                            B                         7 --
            |-------------------------------|
 9          B                              -N           2 v
     |------------|                |------------|
10   -G           B                -G           B       4 >
  |-----|      |-----|          |-----|      |-----|
11 * --G  N    * --G  N         * --G  N     * --G  N   5 >
12   G    X      G    X           G    X       G    X   11 --
     X                            X
```

INVALID. C.E.: (-N&B&G)

9-2. h)

```
 1                    * (D > H) > P                     P
 2                    * D > -(F v G)                    P
 3                       * F v H                        P
 4                      * -(D > P)                      -C
 5                          D                           4 ->
 6                          -P                          4 ->
               |-------------------------|
 7         * -(D > H)                    P              1 >
 8             D                         X              7 ->
 9             -H                                       7 ->
     |--------------|
10   -D        * -(F v G)                               2 >
11   X             -F                                   10 -v
12                 -G                                   10 -v
              |-----------|
13            F           H                             3 v
              X           X
```

VALID

9-2. i)

```
 1                   * I > (J > K)                      P
 2                 * -[(I > J) > K]                     -C
 3                     * I > J                          2 ->
 4                       -K                             2 ->
       |--------------------------|
 5     -I                    * J > K                    1 >
   |-------|        |-------------------|
 6 -I      J        -I                  J               3 >
                |------|            |------|
                -J     K            -J     K            5 >
                       X            X      X
```

INVALID. C.E.: (-K&-I&J); (-K&-I&-J)

9-2. j)

```
 1                  * F & [P v -(D > F)]              P
 2                   * D v -(F v K)                   P
 3                    * -(K > D)                      -C
 4                        F                           1 &
 5                    P v -(D > F)                    1 &
 6                        K                           3 ->
 7                        -D                          3 ->
            |--------------^--------------|
 8          D                   * -(F v K)            2 v
 9          X                        -F               8 -v
10                                   -K               8 -v
                                     X
```

VALID

9-2. k)

```
 1                  * (T & G) v (G & -M)              P
 2                     * -(T & -M)                    -C
            |--------------^--------------|
 3       * T & G                     * G & -M         1 v
 4          T                           G             3 &
 5          G                          -M             3 &
     |------^------|            |------^------|
 6  -T          * --M          -T          * --M      2 -&
 7   X              M                           M     6 --
                                                X
```

INVALID. C.E.: (T&G&M); (G&-M&-T)

9-2. l)

```
 1                * (H<->-Q)<->(H<->-M)               P
 2              * -(H > [Q v -(-Q & M)])              -C
 3                         H                          2 ->
 4                 * -[Q v -(-Q & M)]                 2 ->
 5                        -Q                          4 -v
 6                   * --(-Q & M)                     4 -v
 7                     * -Q & M                       6 --
 8                        -Q                          7 &
 9                         M                          7 &
           |---------------^---------------|
10      * H<->-Q                    * -(H<->-Q)       1 <>
11      * H<->-M                      -(H<->-M)       1 <>
     |------^------|            |------^------|
12   H             -H           H             -H      10 <>;-<>
13  -Q             -Q        * --Q            -Q      10 <>;-<>
 |---^---|          X           |              X
14  H     -H                    |                     11 <>
15 -M    --M                    |                     11 <>
16  X     X                     Q                     13 --
                                X
```

VALID

9-2. m)

```
 1                        * Q<->M                                  P
 2                 * -[(H<->-Q)<->(H<->-M)]                        -C
           |-----------------------^-----------------------|
 3         Q                                              -Q       1 <>
 4         M                                              -M       1 <>
    |------^------|                              |------^------|
 5 * H<->-Q    * -(H<->-Q)               * H<->-Q     * -(H<->-Q)  2 -<>
 6 * -(H<->-M)   * H<->-M                * -(H<->-M)    * H<->-M   2 -<>
  |---^---|     |---^---|                |---^---|     |---^---|
 7 H      -H    H      -H                H      -H     H      -H   5 <>;-<>
 8 -Q  * --Q  * --Q    -Q               -Q   * --Q * --Q      -Q   5 <>;-<>
 9 X      Q     Q       X                |      Q      Q       |   8 --
       |--^--| |--^--|                |--^--|   X      X    |--^--|
10     H   -H  H   -H                 H   -H                H   -H   6 -<>;<>
11   --M  -M  -M  --M              * --M  -M               -M * --M  6 -<>;<>
12     X   X   X   X                  M   X                 X    M   11 --
                                      X                          X
```

VALID

9-2. n)

```
 1                     * H > [D > (B v P)]                 P
 2                         * D v P                         P
 3                        * -(H > P)                       -C
 4                             H                           3 ->
 5                            -P                           3 ->
      |-----------------------^-----------------------|
 6   -H                          * D > (B v P)             1 >
      X                    |-----------^-----------|
 7                         D                       P       2 v
               |-----------^---|                   X
 8            -D             * B v P                       6 >
               X             |---^---|
 9                           B       P                     8 v
                                     X
```

INVALID. C.E.: (H&-P&D&B)

9-2. o)

```
 1                   * -[-K & -(-A & -B)]                  P
 2                   * -[(A > K) & (K > B)]                -C
          |-------------------^-------------------|
 3      * --K                         * --(-A & -B)        1 -&
 4        K                           *  -A & -B           3 --
 5        |                                -A              4 &
 6        |                                -B              4 &
    |-----^-----|                 |--------^--------|
 7 * -(A > K)  * -(K > B)      * -(A > K)      * -(K > B)  2 -&
 8    A             K             A                 K      7 ->
 9   -K            -B            -K                -B      7 ->
      X                           X
```

INVALID. C.E.: (-A&K&-B)

9-2. p)

```
 1                 * (G v A) > (H > B)                      P
 2                 * [H > (H & B)] > K                      P
 3                   * -(G > K)                             -C
 4                       G                                  3 ->
 5                      -K                                  3 ->
           |-------------^-------------|
 6   * -(G v A)                     * H > B                 1 >
 7       -G                            |                    6 -v
 8       -A                       |----^-------|            6 -v
 9        X              * -[H > (H & B)]      K            2 >
10                               H             X            9 ->
11                          * -(H & B)                      9 ->
                          |----^----|
12                        -H        B                       6 >
                           X     |--^--|
13                              -H    -B                    11 -&
                                 X     X
VALID
```

9-2. q)

```
 1                    * D v (M > J)                         P
 2               * [M > (M & J)] > (P v K)                  P
 3                 * (P > D) & (K > F)                      P
 4                    * -(D v F)                            -C
 5                      * P > D                             3 &
 6                      * K > F                             3 &
 7                        -D                                4 -v
 8                        -F                                4 -v
     |---------------------^----------------|
 9   D                                 * M > J              1 v
     X                         |----------^----------|
10                    * -[M > (M & J)]            * P v K   2 >
11                          M                        |      10 ->
12                      * -(M & J)                   |      10 ->
                       |-----^-----|           |-----^----|
13                    -P           D          -P          D 5 >
                   |--^--|         X        |--^--|       X
14                -K     F                 -K     F         6 >
                 |^-|    X               |--^-|   X
15              -M  J                   -M    J             9 >
                 X |--|                  |    |
16                -M  -J                |-|  |-|            12 -&
17                 X   X                P K  P K            10 v
                                        X X  X X
VALID
```

9-2. r)

```
 1        • [(F & -B) v Q] > [A & (S v T)]                  P
 2                • F & -(S & A)                            P
 3                  * S v A                                 P
 4               • -[B > (A > S)]                           -C
 5                     F                                    2 &
 6                 • -(S & A)                               2 &
 7                     B                                    4 ->
 8                 • -(A > S)                               4 ->
 9                     A                                    8 ->
10                    -S                                    8 ->
           |-----------^-------------|
11  • -[(F & -B) v Q]          * A & (S v T)                1 >
12     • -(F & -B)                   A                      11 -v;&
13          -Q                     * S v T                  11 -v;&
      |------^-----|          |------^------|
14    S            A          S             A               3 v
      X        |---^---|      X        |----^----|
15            -S      -A              -S         -A         6 -&
            |-^--|     X            |--^--|       X
16         -F* --B                  S     T                 12 -&;13 v
17          X   B                   X                       16 --
```

INVALID. C.E.: (F&B&A&-S&-Q); (F&B&A&-S&T)

9-2. s)

```
 1                    * F > (K v B)                         P
 2                * (-F v G) & (-G v -K)                    P
 3                    * -(F > B)                            -C
 4                     * -F v G                             2 &
 5                     * -G v -K                            2 &
 6                         F                                3 ->
 7                        -B                                3 ->
      |--------------------^--------------|
 8   -F                                * K v B              1 >
      X                    |-------------^------------|
 9                        -F                          G     4 v
                           X                    |-----^-----|
10                                             -G          -K     5 v
                                                X        |-^--|
11                                                       K    B   8 v
                                                         X    X
VALID
```

9-2. t)

```
1                  ● F<->[-J v (C & T)]                 P
2                    ● A v (C<->O)                      P
3                   ● -[F > (J > -O)]                   P
4                   ● -[F<->(T & A)]                    -C
5                          F                            3 ->
6                    * -(J > -O)                        3 ->
7                          J                            6 ->
8                       * --O                           6 ->
9                          O                            8 --
              |------------^------------|
10            F                         -F              1 <>
11      * -J v (C & T)           -[-J v (C & T)]        1 <>
    |---------^---------|               X
12  A                 C<->O                             2 v
 |--^---|          |---^----|
13  F    -F          F       -F                         4 -<>
14*-(T&A) T&A   * -(T & A)    T & A                     4 -<>
 |--^--|   X     |--^---|      X
15 -J  *C&T      -J   * C & T                           11 v
16 X    C         X      C                              15 &
17      T                T                              15 &
   |--^--|           |--^---|
18 -T   -A           -T     -A                          14 -&
   X    X            X   |--^--|
                         C     -C                       12 <>
                         O     -O                       12 <>
INVALID.  C.E.:  F&J&O&C&T     X
```

9-2. u)

```
1                   * Q<->-(A & F)                       P
2                   * -(M v A) > -H                      P
3                   * -(Q & A) v F                       P
4                  * -[Q > (H > M)]                      -C
5                          Q                             4 ->
6                     * -(H > M)                         4 ->
7                          H                             6 ->
8                          -M                            6 ->
              |------------^------------|
9             Q                         -Q               1 <>
10        * -(A & F)                 --(A & F)           1 <>
         |------^--------|               X
11   * --(M v A)         -H                              2 >
12     * M v A            X                              11 --
        |---^-------|
13  * -(Q & A)      F                                    3 v
     |--^---|     |--|
14   -A     -F   -A  -F                                  10 -&
   |-^-|   |^-|  |-^-| X
15 M   A   M  A  M   A                                   12 v
   X   X   X |--| X  X
16          -Q -A                                        13 -&
VALID       X  X
```


9-2. v)

```
1                  ● (I & -T) > P                        P
2                    * -A > -T                           P
3                    * -T v C                            P
4                     ● C > D                            P
5              ● -(-P > [I > (D & A)])                   -C
6                       -P                               5 ->
7                 ● -[I > (D & A)]                       5 ->
8                        I                               7 ->
9                   ● -(D & A)                           7 ->
           |-------------^-------------|
10     * -(I & -T)                     P                 1 >
    |-------^-------|                  X
11  -I            * --T                                  10 -&
12  X                T                                   11 --
               |-----^-----|
13           * --A         -T                            2 >
14              A          X                             13 --
           |----^----|
15         -T        C                                   3 v
           X     |---^---|
16              -C       D                               4 >
                X    |---^---|
                    -D       -A                          9 -&
VALID               X        X
```

9-3. a)

```
1            ● -[-(-F & G) v -(F & G)]                   -S
2                ● --(-F & G)                            1 -v
3                ● --(F & G)                             1 -v
4                  * -F & G                              2 --
5                  * F & G                               3 --
6                     -F                                 4 &
7                     G                                  4 &
8                     F                                  5 &
9                     G                                  5 &
LOGICAL TRUTH         X
```

9-3. b)

```
1         * -{[H > (O > N)] > [(H & O) > N]}             -S
2                 * H > (O > N)                          1 ->
3               * -[(H & O) > N]                         1 ->
4                   * H & O                              3 ->
5                     -N                                 3 ->
6                     H                                  4 &
7                     O                                  4 &
         |------------^------------|
8        -H                     * O > N                  2 >
         X            |------------^------------|
9                     -O                        N        8 >
LOGICAL TRUTH         X                         X
```

9-3. c)

```
1              * -{[-S & (G v K)] v [-G > (S v K)]}           -S
2                    * -(-S & (G v K))                        1 -v
3                    * -(-G > (S v K))                        1 -v
4                          -G                                 3 ->
5                       * -(S v K)                            3 ->
6                          -S                                 5 -v
7                          -K                                 5 -v
           |-----------------^-----------------|
8       * --S                              * -(G v K)         2 -&
9          S                                   |              8 --
10         X                                  -G              8 -v
11                                            -K              8 -v
```

NOT A LOGICAL TRUTH. C.E.: (-G&-S&-K)

9-3. d)

```
1          * -{[(F v G) > (P & Q)] > (-Q > -F)}        -S
2                * (F v G) > (P & Q)                   1 ->
3                  * -(-Q > -F)                        1 ->
4                       -Q                             3 ->
5                     * --F                            3 ->
6                        F                             5 --
           |-------------^-------------|
7     * -(F v G)                    * P & Q            2 >
8          -F                          P               7 -v;&
9          -G                          Q               7 -v;&
           X                           X
```

LOGICAL TRUTH

9-3. e)

```
1              * -{[L v (M & N)] > [(L v M) & N]}                -S
2                     * L v (M & N)                              1 ->
3                   * -[(L v M) & N]                             1 ->
             |-----------------^-----------------|
4            L                               * M & N             2 v
5            |                                 M                 4 &
6            |                                 N                 4 &
        |----^----|                   |--------^--------|
7   * -(L v M)   -N             * -(L v M)              -N       3 -&
8        -L                          -L                  X       7 -v
9        -M                          -M                          7 -v
         X                           X
```

NOT A LOGICAL TRUTH. C.E.: (L&-N)

9-3. f)

```
1       * -{[(K & M) & P] > [(-K & P) v (K v -M)]}       -S
2                  * (K & M) & P                         1 ->
3            * -[(-K & P) v (K v -M)]                    1 ->
4                    * K & M                             2 &
5                       P                                2 &
6                   -(-K & P)                            3 -v
7                 * -(K v -M)                            3 -v
8                       K                                4 &
9                       M                                4 &
10                     -K                                7 -v
11                     --M                               7 -v
                        X
```

LOGICAL TRUTH

9-3. g)

```
1        * -{[L > (M > N)] > [(L > M) > (L > N)]}         -S
2                 * L > (M > N)                           1 ->
3            * -[(L > M) > (L > N)]                       1 ->
4                    * L > M                              3 ->
5                  * -(L > N)                             3 ->
6                       L                                 5 ->
7                      -N                                 5 ->
       |----------------^----------------|
8     -L                              * M > N             2 >
       X                   |-------------^-------------|
9                         -L                           M  4 >
                           X                  |--------^--------|
10                                           -M                 N  8 >
                                              X                 X
```

LOGICAL TRUTH

9-3. h)

```
1      * -{[(S v T) > F] > [((F v G) > H) > (S > H)]}      -S
2                  * (S v T) > F                           1 ->
3           * -(((F v G) > H) > (S > H))                   1 ->
4                  * (F v G) > H                           3 ->
5                    * -(S > H)                            3 ->
6                         S                                5 ->
7                        -H                                5 ->
         |----------------^----------------|
8     * -(S v T)                            F              2 >
9          -S                               |              8 -v
10         -T                     |---------^---------|    8 -v
11          X                 * -(F v G)              H    4 >
12                                -F                  X    11 -v
13                                -G                       11 -v
                                   X
```

LOGICAL TRUTH

9-3. i)

```
1              ● -{(I & -J) v [(J & K) v -(K & I)]}             -S
2                         ● -(I & -J)                            1 -v
3                  ● -[(J & K) v -(K & I)]                       1 -v
4                         ● -(J & K)                             3 -v
5                        * --(K & I)                             3 -v
6                          * K & I                               5 --
7                             K                                  6 &
8                             I                                  6 &
              |---------------^---------------|
9            -I                             * --J                2 -&
10            X                               J                  9 --
                                     |--------^--------|
11                                  -J                 -K        4 -&
                                     X                  X
LOGICAL TRUTH
```

9-3. j)

```
1          ● -{[(C & (A v D)) v -(C & F)] v -(A & -G)}          -S
2               * -[(C & (A v D)) v -(C & F)]                    1 -v
3                       ● --(A & -G)                             1 -v
4                     ● -[C & (A v D)]                           2 -v
5                       ● --(C & F)                              2 -v
6                        * A & -G                                3 --
7                        * C & F                                 5 --
8                           A                                    6 &
9                          -G                                    6 &
10                          C                                    7 &
11                          F                                    7 &
          |-----------------^-----------------|
12       -C                               * -(A v D)             4 -&
13        X                                  -A                  12 -v
14                                           -D                  12 -v
                                              X
LOGICAL TRUTH
```

9-4. a)

```
1                 * (A & B) & (-A v -B)                          S
2                        * A & B                                 1 &
3                       * -A v -B                                1 &
4                           A                                    2 &
5                           B                                    2 &
            |---------------^---------------|
6          -A                              -B                    3 v
            X                               X
CONTRADICTION
```


9-4. b)

```
1                 * (F v G) & (-F v -G)                          S
2                        * F v G                                 1 &
3                       * -F v -G                                1 &
            |---------------^---------------|
4           F                               G                    2 v
     |------^------|                 |------^------|
5   -F            -G                -F            -G             3 v
     X                                             X
NOT A CONTRADICTION.  C.E.:  (F&-G); (G&-F)
```

9-4. c)

```
1                  ● (F v G)<->(-F & -G)                         S
          |-------------------^-------------------|
2       * F v G                             ● -(F v G)           1 <>
3      * -F & -G                           ● -(-F & -G)          1 <>
4         -F                                                     3 &
5         -G                                                     3 &
     |----^----|
6    F         G                                                 2 v
7    X         X                                -F               2 -v
8                                               -G               2 -v
                                         |------^------|
9                                      ● --F         ● --G       3 -&
10                                        F             G        9 --
CONTRADICTION                             X             X
```

9-4. d)

```
1              * [I v (J & K)] > [(I v J) & K]                   S
                |-------------^-------------|
2       ● -[I v (J & K)]               ● (I v J) & K             1 >
3              -I                         * I v J                2 -v;&
4          ● -(J & K)                        K                   2 -v;&
        |------^------|              |-------^-------|
5      -J            -K              I               J           4 -&;3 v
 NOT A CONTRADICTION.  C.E.: (-I&-J); (-I&-K); (K&I); (K&J)
```

9-4. e)

```
1              * [(H & F) > C] & -[H > (F > C)]                  S
2                       * (H & F) > C                            1 &
3                      * -[H > (F > C)]                          1 &
4                             H                                  3 ->
5                         * -(F > C)                             3 ->
6                             F                                  5 ->
7                            -C                                  5 ->
                 |------------^------------|
8            * -(H & F)                    C                     2 >
         |-------^-------|                 X
9       -H              -F                                       8 -&
         X               X      CONTRADICTION
```

9-4. f)

```
1              * -{[B & (M v -P)] > [P > (-M v -B)]}          S
2                      * B & (M v -P)                         1 ->
3                    * -(P > (-M v -B))                       1 ->
4                            B                                2 &
5                        * M v -P                             2 &
6                            P                                3 ->
7                       * -(-M v -B)                          3 ->
8                          * --M                              7 -v
9                          * --B                              7 -v
10                           M                                8 --
11                           B                                9 --
              |--------------^--------------|
12            M                            -P                 5 v
                                            X
```

NOT A CONTRADICTION. C.E.: (B&P&M)

9-4. g)

```
1                      * (A<->B)<->(-A<->B)                         S
              |-----------------^-----------------|
2          * A<->B                          * -(A<->B)              1 <>
3         * -A<->B                         * -(-A<->B)              1 <>
       |------^------|                  |------^------|
4      A             -A                 A             -A            2 <>;-<>
5      B             -B                -B              B            2 <>;-<>
   |---^---|      |---^---|         |---^---|     |---^---|
6  -A     --A    -A    * --A       -A     --A    -A   * --A         3 <>;-<>
7   B      -B     B      -B        -B      B     -B      B          3 <>;-<>
8   X       X     X       A         X      X      X      A          6 --
                          X                              X
```

CONTRADICTION

9-4. h)

```
1               * [(-F v Q) v (P & Q)] & -(-Q > -F)            S
2                   * (-F v Q) v (P & Q)                        1 &
3                      * -(-Q > -F)                             1 &
4                           -Q                                  3 ->
5                         * --F                                 3 ->
6                           F                                   5 --
               |------------^------------|
7          * -F v Q                   * P & Q                   2 v
       |-------^-------|                 |
8     -F               Q                 |                      7 v
9      X               X                 P                      7 &
10                                       Q                      7 &
                                         X
```

CONTRADICTION

9-4. i)

```
1           * [K > (D > P)] & [(-K v D) & -(K > P)]            S
2                   * K > (D > P)                              1 &
3               * (-K v D) & -(K > P)                          1 &
4                     * -K v D                                 3 &
5                    * -(K > P)                                3 &
6                        K                                     5 ->
7                       -P                                     5 ->
     |-------------------^-------------------|
8   -K                                   * D > P               2 >
     X                        |----------^----------|
9                            -K                     D          4 v
                              X              |------^------|
10                                          -D             P   8 >
```

CONTRADICTION

```
                                             X             X
```

9-4. j)

```
1            * [(-G v Q) & (K > G)] & -(P v -K)              S
2                 * (-G v Q) & (K > G)                       1 &
3                     * -(P v -K)                            1 &
4                       * -G v Q                             2 &
5                       * K > G                              2 &
6                          -P                                3 -v
7                        * --K                               3 -v
8                           K                                7 --
          |-----------------^-----------------|
9        -G                                   Q              4 v
    |-----^-----|                     |-------^-------|
10 -K           G                    -K               G      5 >
    X           X                     X
```

NOT A CONTRADICTION. C.E.: (-P&K&Q&G)

9-5. From exercise 4-3, we saw that two sentences X and Y are logically equivalent if and only if the two arguments "X. Therefore Y" and "Y. Therefore X" are both valid. Since we can test the validity of arguments using truth trees, we can formulate a new way of testing logical equivalence with trees by using this information. To determine whether X and Y are logically equivalent, it is sufficient to test both of the arguments X/Y and Y/X for validity. Note that with this new method, we need two truth trees.

9-6. a)

```
1                  * -[(A > -A) <-> -A]                      -S
            |-------------^-------------|
2       * A > -A                  * -(A > -A)                1 -<>
3        * --A                        -A                     1 -<>
4          A                           |                     3 --
     |-----^-----|                     |
5   -A           -A                    |                     2 >
6    X            X                    A                     2 ->
7                                     --A                    2 ->
```

LOGICALLY EQUIVALENT

```
                                       X
```

9-6. b)

```
1                * -[-(I v J)<->(-I & -J)]               -S
                 |------------^------------|
2         * -(I v J)                  * --(I v J)        1 -<>
3       * -(-I & -J)                   * -I & -J         1 -<>
4            -I                         * I v J          2 -v;--
5            -J                             |            2 -v
        |-----^-----|                       |
6     * --I       * --J                     |            3 -&
7       I           J                       |            6 --
8       X           X                      -I            3 &
9                                          -J            3 &
                                     |-----^-----|
10                                   I           J       4 v
                                     X           X
```

LOGICALLY EQUIVALENT

9-6. c)

```
1              * -[(M v -H)<->(-M > H)]                -S
               |-----------^-----------|
2     * M v -H                    * -(M v -H)          1 -<>
3   * -(-M > H)                     * -M > H           1 -<>
4        -M                             -M             3 ->;2 -v
5        -H                           * --H            3 ->;2 -v
6         |                              H             5 --
    |-----^-----|                 |------^------|
7   M          -H               * --M           H      2 v;3 >
    X                              M                   7 --
                                   X
```

NOT LOGICALLY EQUIVALENT. C.E.: (-M&-H); (-M&H)

9-6. d)

```
1                 * -[-(F & P)<->(-F & -P)]                 -S
                  |------------^------------|
2         * -(F & P)                    * --(F & P)         1 -<>
3       * -(-F & -P)                     * -F & -P          1 -<>
4      |------^------|                    * F & P           2 --
5     -F            -P                       -F             2 -&;3 &
6  |--^--|        |--^--|                    -P                 3 &
7 * --F  * --P  * --F  * --P                  F             3 -&;4 &
8   F      P      F      P                    P             7 --;4 &
    X                    X                    X
```

NOT LOGICALLY EQUIVALENT. C.E.: (-F&P)

9-6. e)

```
1            * -{[(D & N) > J]<->[D > (N > J)]}              -S
             |---------------^---------------|
2        * (D & N) > J                * -[(D & N) > J]       1 -<>
3      * -[D > (N > J)]                 * D > (N > J)        1 -<>
4            D                                |              3 ->
5        * -(N > J)                           |              3 ->
6            N                                |              5 ->
7           -J                                |              5 ->
       |-----^-----|                          |
8   * -(D & N)     J                          |              2 >
   |---^---|       X                          |
9  -D      -N                                 |              8 -&
10  X       X                              * D & N           2 ->
11                                            -J             2 ->
12                                             D             10 &
13                                             N             10 &
                                       |-------^-------|
14                                    -D             * N > J      3 >
                                       X            |---^---|
15                                                 -N       J    14 >
                                                    X       X
```

LOGICALLY EQUIVALENT

9-6. f)

```
1           * -{[(I > Q) > D]<->[I > (Q > D)]}            -S
            |---------------^---------------|
2       * (I > Q) > D                 * -[(I > Q) > D]    1 -<>
3     * -[I > (Q > D)]                  * I > (Q > D)     1 -<>
4           I                                 |           3 ->
5       * -(Q > D)                            |           3 ->
6           Q                                 |           5 ->
7          -D                                 |           5 ->
      |-----^-----|                           |
8   * -(I > Q)    D                           |           2 >
9       I         X                           |           8 ->
10     -Q                                     |           8 ->
11      X                                  * I > Q        2 ->
12                                            -D          2 ->
                                       |------^------|
13                                    -I           * Q > D      3 >
                                   |--^--|        |--^--|
14                                -I     Q       -Q     D       11 >;13 >
                                                         X
```

NOT LOGICALLY EQUIVALENT. C.E.: (-D&-I&Q); (-D&-Q)

9-6. g)

```
1               • -{[L & (S v T)]<->[(L & S) v (L & T)]}                -S
                  |--------------------------------------|
2       * L & (S v T)                     • -(L & (S v T))             1 -<>
3    • -[(L & S) v (L & T)]              • (L & S) v (L & T)           1 -<>
4            L                          |---------^--------|           2 &
5          * S v T                     -L              • -(S v T)      2 &;-&
6         • -(L & S)                 |--^--|               |           3 -v
7         • -(L & T)          * L & S*L & T                |           3 -v;v
       |--------^--------|           |     |               |
8      S                 T           |     |               |           5 v
   |---^---|         |---^---|       |     |               |
9  -L      -S       -L       -S      |     |               |           6 -&
   X        X        X     |-^-|     |     |               |
10                        -L   -T    |     L               |           7 -&;&
11                         X    X    L     T               |           7 &
12                                   S     X               |           7 &
13                                   X                    -S           5 -v
14                                                        -T           5 -v
                                                      |----^----|
15                                               * L & S     * L & T   3 v
16                                                  L           L      15 &
17                                                  S           T      15 &
                                                    X           X
```

LOGICALLY EQUIVALENT

9-6. h)

```
1              * -{[H v -(-P v -Q)]<->[(H v P) & (H v Q)]}               -S
                 |-----------------------^---------------|
2         * H v -(-P v -Q)                     -(H v -(-P v -Q))        1 -<>
3      * -[(H v P) & (H v Q)]                  (H v P) & (H v Q)        1 -<>
          |--------^--------|                          |
4   * -(H v P)         * -(H v Q)                     [1]               3 -&
5        -H                 -H                                          4 -v
6        -P                 -Q                                          4 -v
   |-----^---|        |-----^---|
7  H * -(-P v -Q)     H * -(-P v -Q)                                    2 v
8  X     * --P        X     * --P                                       7 -v
9          --Q              * --Q                                       7 -v
10          P                  P                                        8 --
11          X                  Q                                        9 --
                               X
```

FOR COMPLETION OF THIS TREE, SEE [1] BELOW.

[1] (9-6. h CONTINUED)

```
2                  * -(H v -(-P v -Q))                 1 -<>
3                  * (H v P) & (H v Q)                 1 -<>
4                          -H                          2 -v
5                     * --(-P v -Q)                    2 -v
6                        * H v P                       3 &
7                        * H v Q                       3 &
8                       * -P v -Q                      5 --
      |-----------------------^-----------------|
9     H                                         P      6 v
      X                             |-----------^----------|
10                                  H                      Q          7 v
                                    X               |------^-----|
12                                                 -P           -Q    8 v
                                                    X            X
```

9-7.

a) Yes. According to the definition (C1), a sentence of SL is consistent if and only if it is not a contradiction. But, no logical truth con be a contradiction; so, all logical truths are consistent.

b) Since a contradiction is defined as a sentence which cannot be true under any possible assignment of truth values to atomic sentence letters, any sentence which has at least one assignment of truth values to atomic sentence letters which makes it true is not a contradiction. But, if the sentence is not a contradiction, then it is consistent (C1). Likewise, if a sentence is consistent according to (C1), then it is not a contradiction; thus, there is at least one assignment of truth values to atomic sentences which makes the sentence true.

c) (C2) essentially states the fact that the set {X,Y,Z} is consistent if and only if there is an assignment of truth values to atomic sentences which makes X and Y and Z all true. But, wherever this is the case, there is an assignment which makes X&Y&Z true. And, according to (C1), any sentence o SL is consistent if it is not a contradiction. If X&Y&Z is true in an assignment, then it is not a contradiction. Therefore, a set of sentences is consistent according to (C2) if and only if the conjunction of its members is consistent according to (C1).

d) No, the given set is not consistent. According to (C2), a set of sentences is consistent if and only if there is an assignment of truth values to sentence letters which makes all members of the set true. But, any assignment which makes -A true will make --a false, and vice versa.

e) Any infinite set of sentences which does not include a sentence X and its negation -X (or some sentence or sentences which imply -X) is a consistent set. Here is one example: {A1, A2, A3, ...}.

9-7. f1)

```
1                          * P v S                         S
2                          * P > S                         S
           |------------------^------------------|
3          P                                     S         1 v
   |-------^-------|                   |---------^--------|
4 -P               S                  -P                  S    2 >
   X
```

CONSISTENT. MODEL: (P&S); (S&-P)

9-7. f2)

```
1                      * (-F > S) > F                 S
2                            F                        S
3                            S                        S
                 |-----------^------------|
4          * -(-F > S)                    F           1 >
5                 -F                                  4 ->
6                 -S                                  4 ->
                   X
```

CONSISTENT. MODEL: (F&S)

9-7. f3)

```
1                  * K & [(-K & H) v H]               S
2                            K                        1 &
3                     * (-K & H) v H                  1 &
                 |-----------^------------|
4            * -K & H                     H           3 v
5                 -K                                  4 &
6                  H                                  4 &
                   X
```

CONSISTENT. MODEL: (K&H)

9-7. f4)

```
1                          * N v B                          S
2                         * N v -B                          S
3                         * -N v B                          S
              |----------------^---------------|
4             N                                B            1 v
      |-------^------|              |----------^----------|
5     N             -B              N                    -B 2 v
  |---^---|      |---^---|      |---^---|                 X
6 -N      B     -N       B     -N       B                   3 v
  X                             X
```

CONSISTENT. MODEL: (N&B)

g) Recall the definition of validity given in Chapter 4 - i.e., an argument is valid if and only if any assignment which makes the premises all true also makes the conclusion true. Suppose we have an argument X/Y where X is the conjunction of the premises. This definition tells us that any time X is true, Y must be true. But, if X entails the truth of Y, then the set {X,-Y} will be inconsistent, since any assignment which makes X true will also make Y true; and where Y is true, we can never make -Y true also.